Bayesian Poisson log-bilinear models for mortality projections with multiple populations

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joint work with Katrien Antonio and Wilbert Ouburg

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Outline

State of the art: age–period mortality tables

- In a dynamic context, mortality is assumed to be a function of both age x and calendar year t.
- $q_{x,t}$: the probability of dying at age x, for an individual aged x in year t.
- $\mu_{x,t}$: the force of mortality, which is the instantaneous rate of dying, for an individual aged x in year t.
- The relation between the two quantities is $q_{x,t} \approx 1 - \exp\{-\mu_{x,t}\}$.
- Forecasting: simulate $q_{x,t}$ beyond $t = T$, use these in e.g. annuity calculations.

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State of the art: LC type models

- Lee & Carter (LC) (1992, JASA)

$$\log \mu_{x,t} = \alpha_x + \beta_x \cdot \kappa_t$$

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$$\log \mu_{x,t} = \alpha_x + \beta_x \cdot \kappa_t$$

- Renshaw and Haberman (2003, IME) model:

$$\log \mu_{x,t} = \alpha_x + \beta_{x,1} \cdot \kappa_{t,1} + \beta_{x,2} \cdot \kappa_{t,2}$$

  - Avoids the trivial correlation structure of the original LC model.
  - Captures age and time dynamics in a more flexible way.
  - Constraints: for $j = 1, 2$

$$\sum_x \beta_{x,j} = 1, \sum_t \kappa_{t,j} = 0 \text{ and } \sum_t \kappa_{t,1} \cdot \kappa_{t,2} = 0\,.$$

  - Orthogonality constraint $\leftarrow$ not always trivial.

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State of the art: likelihood assumption

- We introduce the following (random) variables:
  - $D_{x,t}$: the number of deaths at age x in year t;
  - $E_{x,t}$: the exposure–at–risk at age x in year t.
- Likelihood assumption:

$$D_{x,t} | E_{x,t} \cdot \mu_{x,t} \sim \text{Poisson}(E_{x,t} \cdot \mu_{x,t}).$$

- Motivation for Poisson assumption:
  - stems from survival likelihood (see Pitacco et al., 2009),
  - account appropriately for uncertainty in number of deaths.

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Why multi–population mortality models?

- Use larger data set at full.
- To account for group membership of populations and countries (e.g. Western Europe).
- More robust modeling by using a larger data set.
- To obtain consistent/coherent projections (e.g. Dutch males and females projections can not diverge).
- To handle basis risk.
- Increasing attention for this setting, see e.g. recent projection by Royal Dutch Actuarial Association (AG-AI) and by the Institute of Actuaries in Belgium (see Antonio, Devolder & Devriendt, 2015).

Multi-population mortality models

Our paper puts focus on:

- LC-2,t model,

$$\log \mu_{x,t}^{(i)} = \underbrace{\alpha_x^{(i)}}_{i-specific} + \underbrace{\beta_{x,1}^{(i)}}_{i-specific} \cdot \underbrace{K_t}_{Common} + \underbrace{\beta_{x,2}^{(i)} \cdot \kappa_t^{(i)}}_{i-specific}$$

- Li & Lee (LL) model (see Li & Lee, 2005, Demography),

$$\log \mu_{x,t}^{(i)} = \underbrace{\alpha_x^{(i)}}_{i-specific} + \underbrace{B_x \cdot K_t}_{Common} + \underbrace{\beta_{x,2}^{(i)} \cdot \kappa_t^{(i)}}_{i-specific}$$

Comparison

- LC-2,t model:
  - Able to apply orthog. constraint,
  - Link populations through a time parameter (K_t),
  - NOT coherent (due to $\beta_{x,1}^{(i)}$),
  - Studied by Danesi et al. (2015).

- LL model:
  - NOT able to apply orthog. constraint (two step approach),
  - Link populations through age and time parameter (B_x, K_t),
  - Coherent (due to B_x),
  - Studied by Li & Lee (2005) and Li (2013).

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Why the Bayesian framework?

Within the Bayesian framework we are able to:

- Unify calibration step + forecast step

  consistent period effects.

- Incorporate parameter uncertainty

  account for all sources of uncertainty in forecasts.

- Handle small populations and missing data (e.g. portfolio data or data on regions of a country).

Bayesian framework: LC-2,t (DAG)

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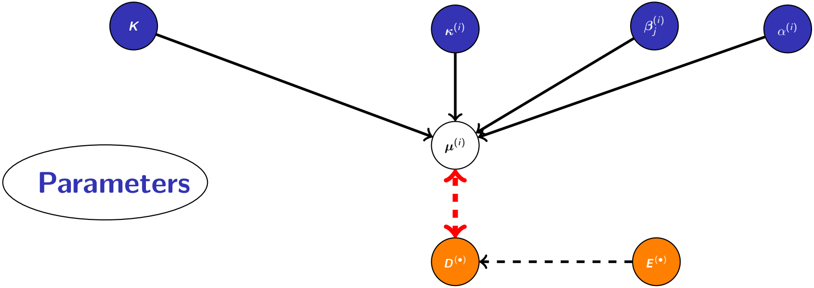
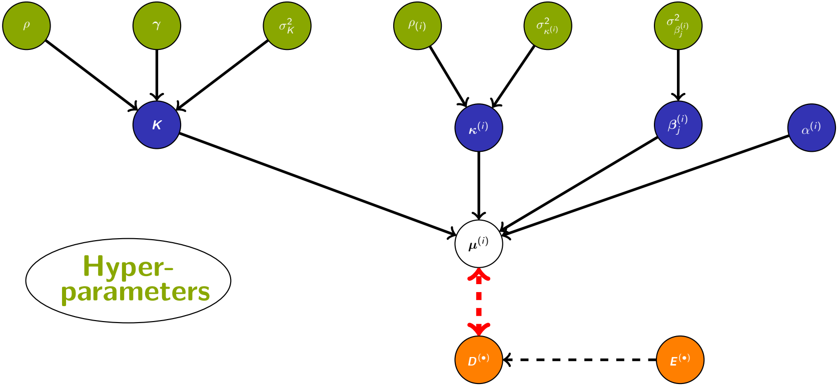
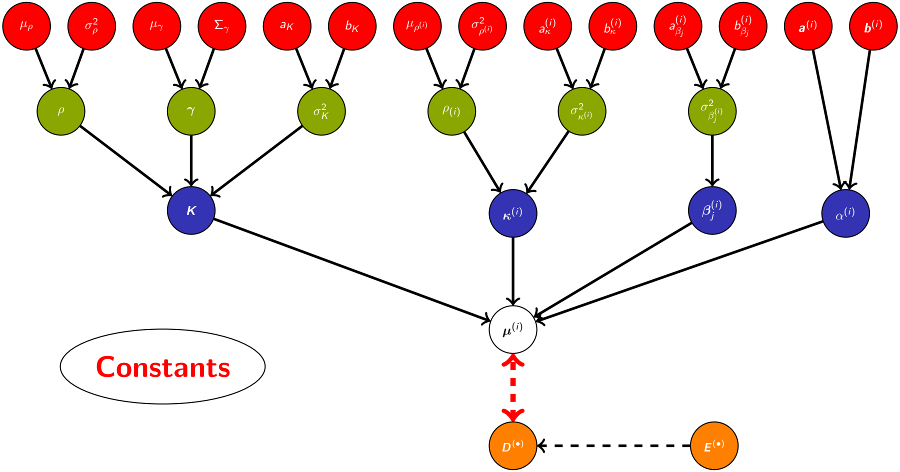


Bayesian framework: LC-2,t (DAG)



Bayesian framework: LC-2,t (DAG)



Bayesian framework: time series prior

- For the K_t we assume $AR(1)$ with linear trend

$$K_t = \eta_t + \rho\left(K_{t-1} - \eta_{t-1}\right) + \epsilon_t \text{ with } \epsilon_t \sim \mathcal{N}\left(0, \sigma_K^2\right),$$

where $\eta_t := \gamma_1 + \gamma_2 t$.

- Thus, prior for K_t

$$K_t | K_1, \ldots, K_{t-1} \sim \mathcal{N}\left(\eta_t + \rho\left(K_{t-1} - \eta_{t-1}\right), \sigma_K^2\right) \quad (1)$$

$$K_1 \sim \mathcal{N}\left(\eta_1, \frac{\sigma_K^2}{1-\rho^2}\right). \quad (2)$$

- A Gaussian Markov Random Field (GMRF) (see Rue et al. (2005))

$$\pi\left(\boldsymbol{K}\right) \sim \mathcal{N}_T\left(\boldsymbol{V}\gamma, \sigma_K^2 \boldsymbol{Q}^{-1}\right) \quad (3)$$

- Similar for all $\kappa_t^{(i)}$ but without a linear trend (see Li & Lee (2005)).

Bayesian framework: time series prior

- Hereby,

$$
\begin{aligned}
\boldsymbol{\gamma} &= (\gamma_1, \gamma_2), \\
\boldsymbol{Q} &= \boldsymbol{Q}_{T,T} = \begin{pmatrix} 1 & -\rho & 0 & \cdots & 0 \\ -\rho & 1+\rho^2 & -\rho & \cdots & 0 \\ 0 & -\rho & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & 1+\rho^2 & -\rho \\ 0 & 0 & \cdots & -\rho & 1 \end{pmatrix}, |\boldsymbol{Q}| = 1-\rho^2, \\
\boldsymbol{V}' &= \begin{pmatrix} 1 & 2 & \dots & T \\ 1 & 1 & \dots & 1 \end{pmatrix},
\end{aligned}
$$

where T is the total number of the years.

Bayesian framework: priors for the age specific parameters

- For the age specific parameters $\beta_{x,j}^{(i)}$ and B_x,

$$\beta_{x,j}^{(i)} \sim \mathcal{N}\left(\frac{1}{M}, \sigma^2_{\beta_j^{(i)}}\right) \text{ and } B_x \sim \mathcal{N}\left(\frac{1}{M}, \sigma^2_B\right),$$

where M is the total number of ages.

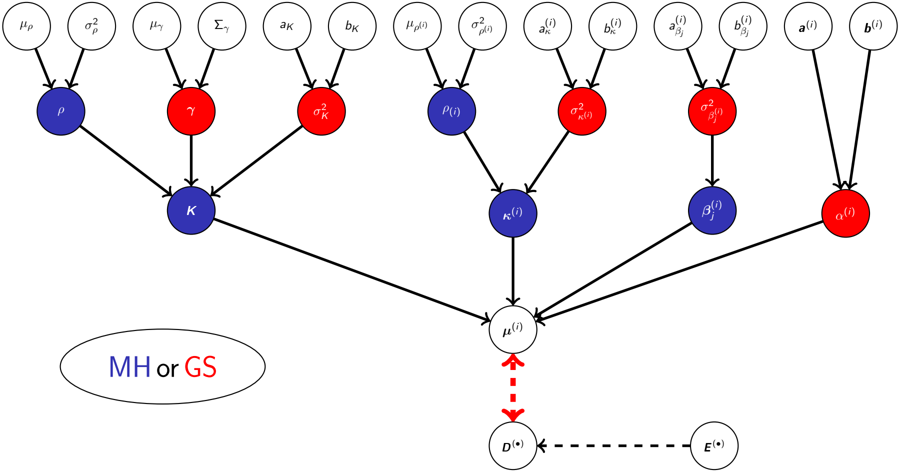
- For the $\alpha_x^{(i)}$ and A_x,

$$\begin{aligned} e_x^{(i)} &= \exp\left(\alpha_x^{(i)}\right) \sim \mathcal{G}\left(a_x^{(i)}, b_x^{(i)}\right). \\ \mathcal{E}_x &= \exp\left(A_x\right) \sim \mathcal{G}\left(a_x, b_x\right). \end{aligned}$$

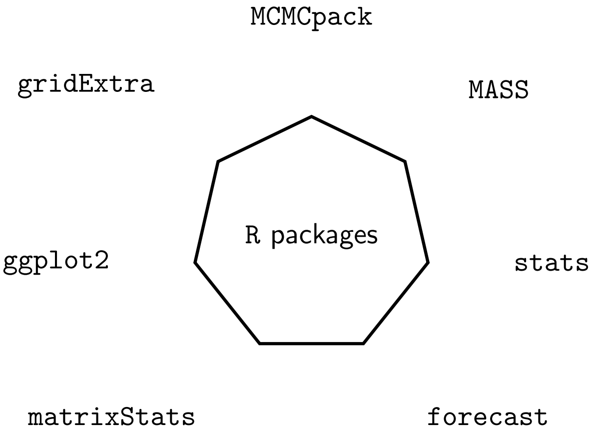
where $a_x^{(i)}$, $b_x^{(i)}$, a_x and b_x are constants.

- See our paper for prior assumptions on the hyperparameters.

Bayesian framework: MH and Gibbs Sampler for LC-2,t



R implementation



Contribution

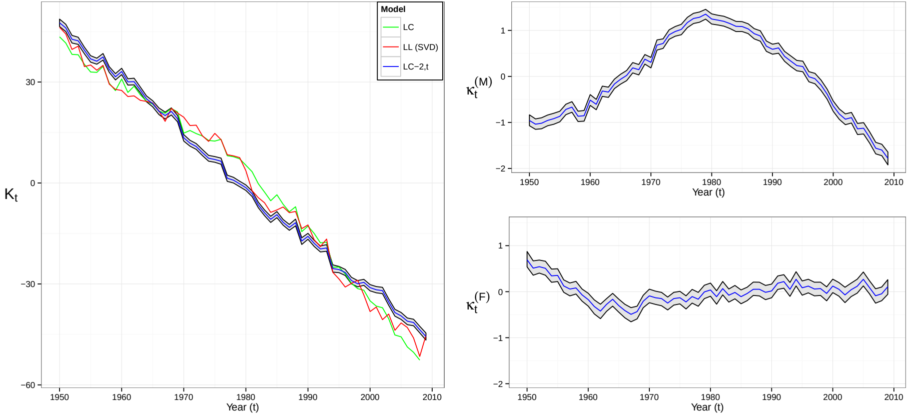
- Full specification of MCMC.
- Two empirical studies including,
  - parameter estimates and uncertainty,
  - convergence checks,
  - mortality forecasts,
  - comparison to LL (SVD) and univariate LC.
- Official institutions (such as CBS, AG-AI or IA|BE) can use this model to challenge their point estimates obtained with MLE.

Empirical Studies

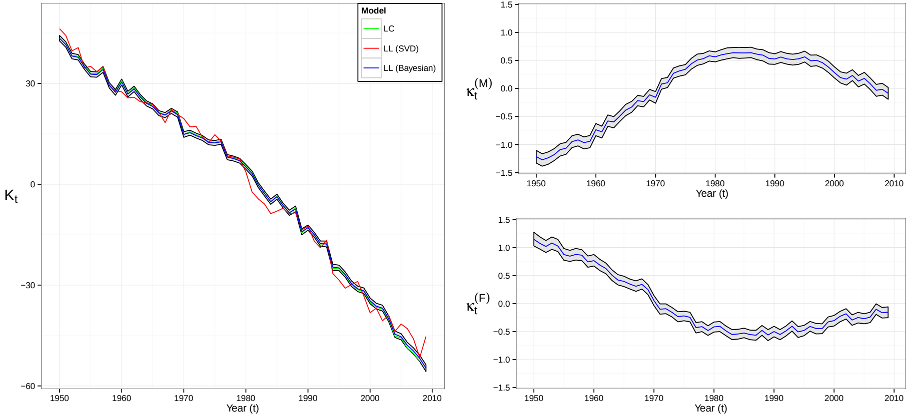
- Empirical study 1:
  - Country: Sweden
  - Gender: males & females,
  - Time period: from 1950 until 2009,
  - Age period: from 0 to 89.
  - Studied also by Li & Lee (2005) and Hyndman et al. (2013).

- Empirical study 2:
  - Country: 14 countries,
  - Gender: females,
  - Time period: from 1975 until 2009,
  - Age period: from 0 to 89.
  - Studied also by AG-AI (2014) and Antonio, Devolder & Devriendt (2015).

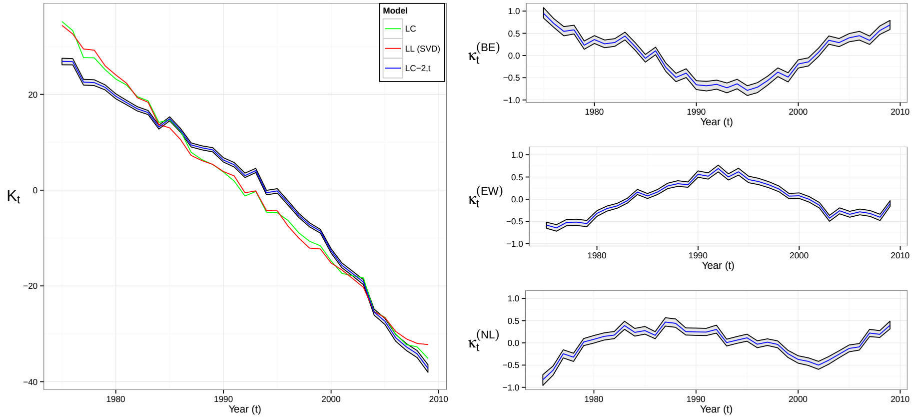
Empirical study: Sweden (LC-2,t model)



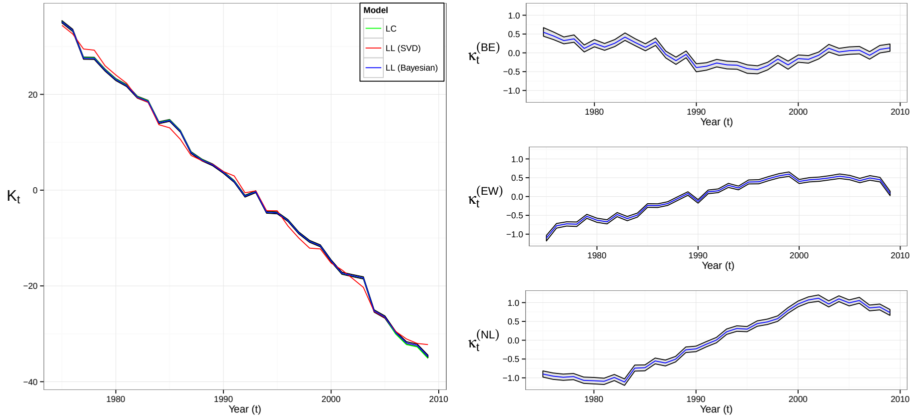
Empirical study: Sweden (LL model)



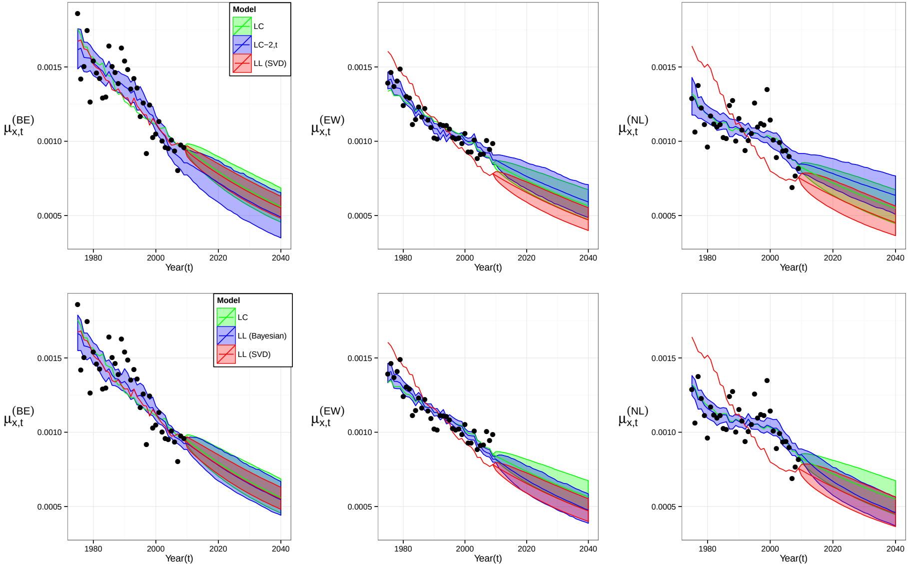
Empirical study: 14 countries (LC-2,t model)



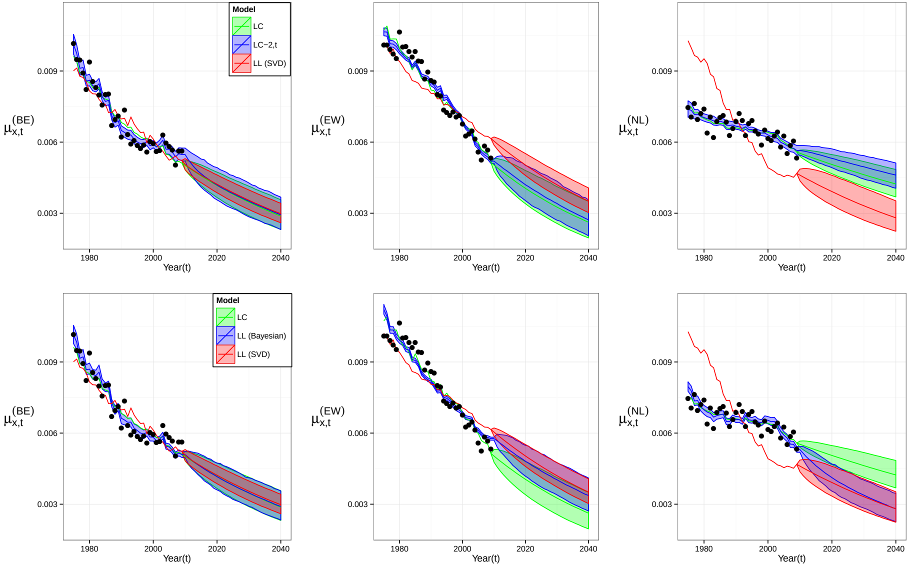
Empirical study: 14 countries (LL model)



14 countries: LC-2,t (top) and LL (bottom) at age 40



14 countries: LC-2,t (top) and LL (bottom) at age 60



Thanks for your time!