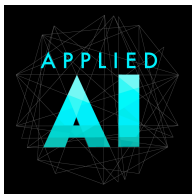


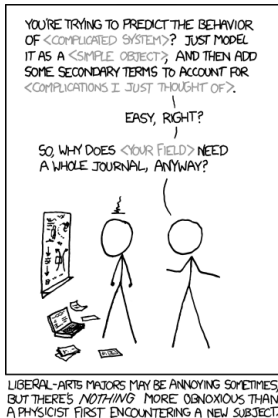
Pricing Mortality Swaps Using R R in Insurance 2015

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Before I Begin...



<https://xkcd.com/793/>

What is a Mortality Swap?

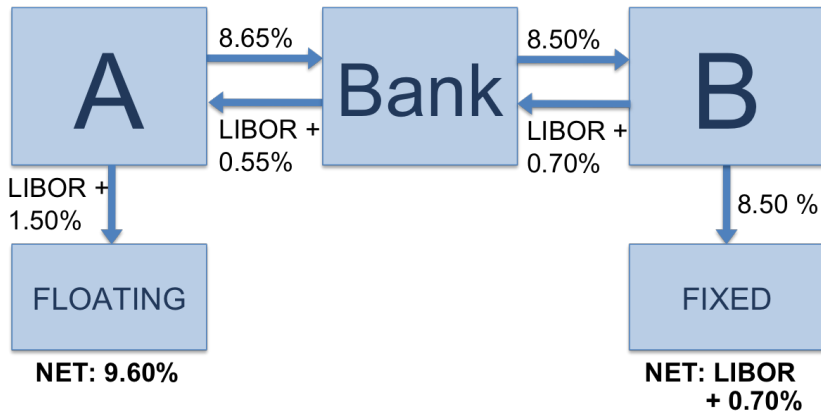
- Seller provides protection against mortality risk
- Portfolio of Life-Contingent Annuities
- Converts the portfolio to Guaranteed Annuities
- Only guaranteeing mortality-adjusted cashflows

Swap Annuity Portfolio

Random portfolio of annuities:

```
##      customerid age MUR   amount
## 1: C97326460  39 350 $100,000
## 2: C99971880  30 175  $80,000
## 3: C65134119  34 225  $50,000
## 4: C35313144  33 200  $90,000
## 5: C17550793  45 200  $30,000
## ---
## 196: C37440555 40 200  $50,000
## 197: C60347677 42 200  $60,000
## 198: C93383952 45 200  $70,000
## 199: C08447895 35 150  $60,000
## 200: C64003360 41 325  $90,000
```

Interest Rate Swaps



Starting point — some generalisable for flexibility

- 1 No consideration of credit risk
- 2 Swap has fixed lifetime
- 3 All annuities have annual payments received at same time
- 4 Fixed cost of capital over lifetime
- 5 All annuitants have undergone underwriting evaluation
- 6 APV calculations require a specific lifetable
- 7 All annuities have a lifetime at least as long as the swap lifetime

Calculating the MUR multiplier

```
lifetable.dt <- fread("lifetable.csv");

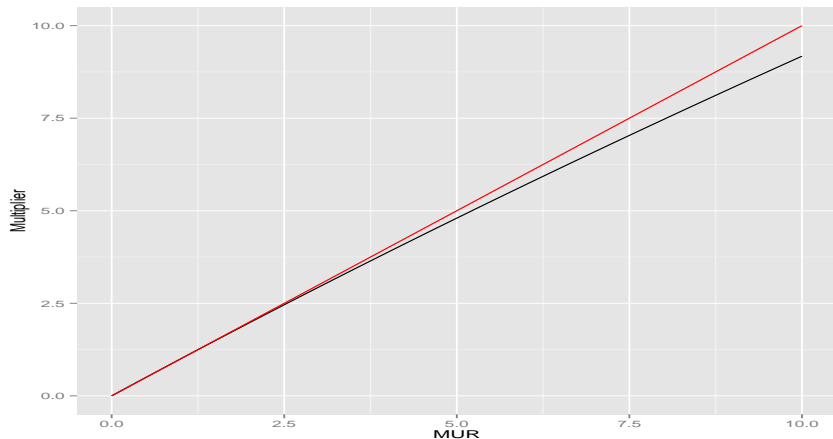
A <- 100000;
qx <- lifetable.dt[age >= 30][age < 50]$qx;
r <- 0.05;

calculate.MUR.multiple.diff <- function(MUR, mult) {
  MUR * price.term.assurance(qx, A = A, r = r, P = 0) -
    price.term.assurance(mult * qx, A = A, r = r, P = 0);
}

MUR.values <- seq(0, 10, by = 0.25);

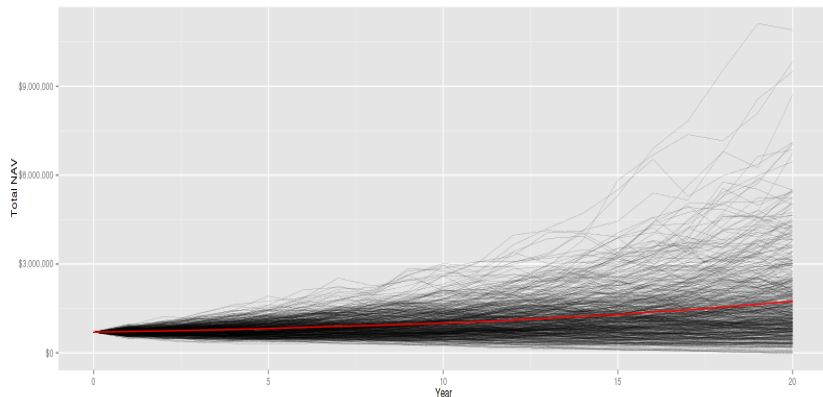
### Determine mortality multiplier that best matches the premium multiplier
MUR.mult <- sapply(MUR.values, function(MUR) {
  optimize(function(mult) abs(calculate.MUR.multiple.diff(MUR, mult)), c(0, 20))$minimum;
});
```

Calculating the MUR multiplier



MonteCarlo Methods

Calculating the NAV of a fund of correlated assets with a yearly drawdown:



Simulation Approach

Simulation example: 10 simulations of 5 years

1 – annuitant still alive

0 – annuitant has deceased

```
##      sim1 sim2 sim3 sim4 sim5 sim6 sim7 sim8 sim9 sim10
## year1  1   1   1   1   1   1   1   1   1   1
## year2  1   1   0   1   1   1   1   1   1   1
## year3  1   1   0   1   1   1   1   1   0   1
## year4  1   1   0   1   0   1   1   1   0   1
## year5  1   1   0   1   0   1   1   1   0   1
```

Running the Simulation

Run the calculation:

```
mortport.dt <- fread("mortswap_portfolio.csv");
lifetable.dt <- fread("lifetable.csv");

n.sim <- 1000;

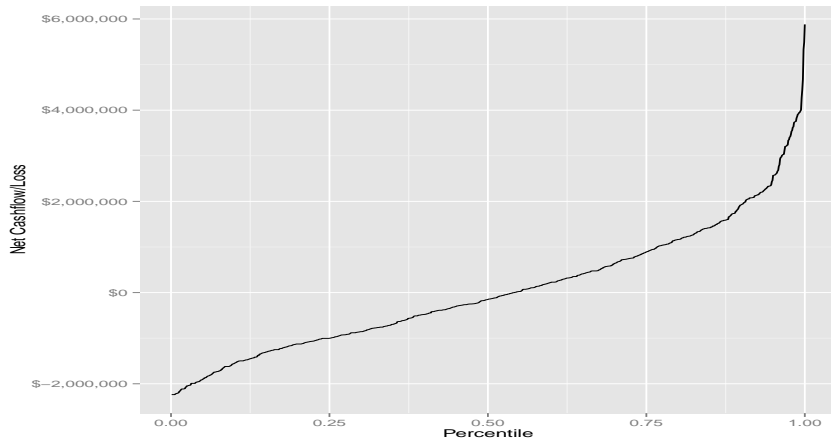
mortswap.value.sim <- calculate.mortality.swap.price(mortport.dt,
                                                    lifetable           = lifetable.dt,
                                                    hedge.apv.cashflows = TRUE,
                                                    interest.rate      = 0.05,
                                                    years.in.force     = 20,
                                                    n.sim              = n.sim,
                                                    return.all.data    = FALSE);

print(dollar_format(largest_with_cents = 1e8)(mortswap.value.sim[1:20]));

## [1] "$748,064.31"    "$461,186.76"    "$2,274,787.66"  "$-1,251,047.55"
## [5] "$1,954,488.45"  "$-1,251,047.55" "$-2,237,366.32" "$1,733,535.87"
## [9] "$1,707,822.63"  "$-1,299,952.30" "$2,139,507.05"  "$-757,888.16"
## [13] "$-309,255.02"   "$-717,420.20"   "$1,927,893.22"  "$1,417,572.90"
## [17] "$1,571,751.87"  "$-739,440.89"   "$-1,990,786.63" "$475,010.30"
```

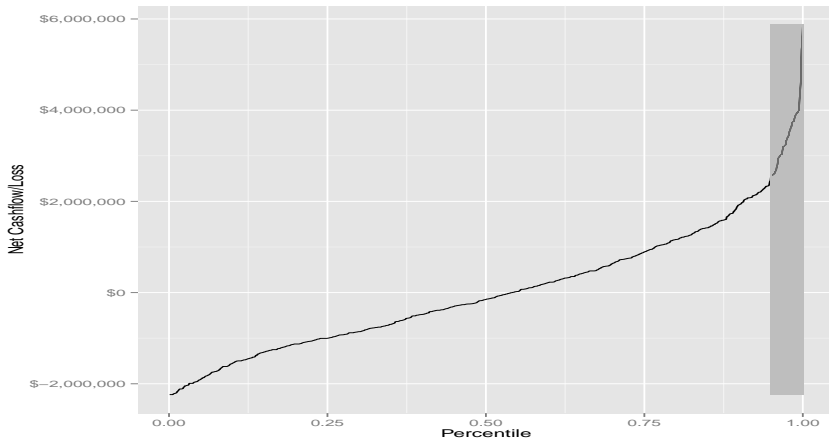
Viewing the Output

Simulated cashflows
(excludes initial premium):



Viewing the Output

Simulated cashflows
(excludes initial premium):



How to price the tail risk?

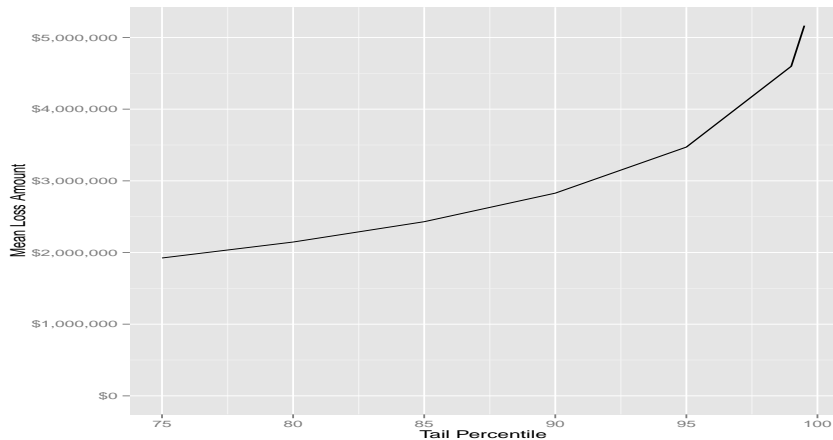
Michael Lewis "*In Nature's Casino*" – NYT Aug 2007

<http://www.nytimes.com/2007/08/26/magazine/26neworleans-t.html?pagewanted=all>

Consensus around $4\times$ expected loss

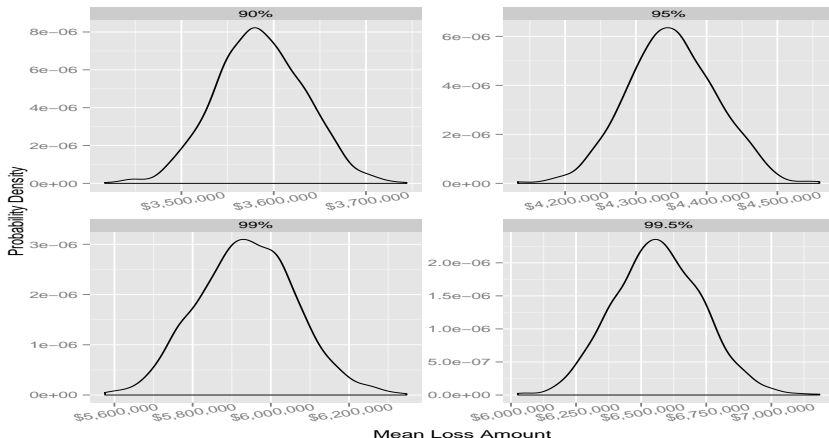
Need to calculate tail averages

Pricing Tail Risk



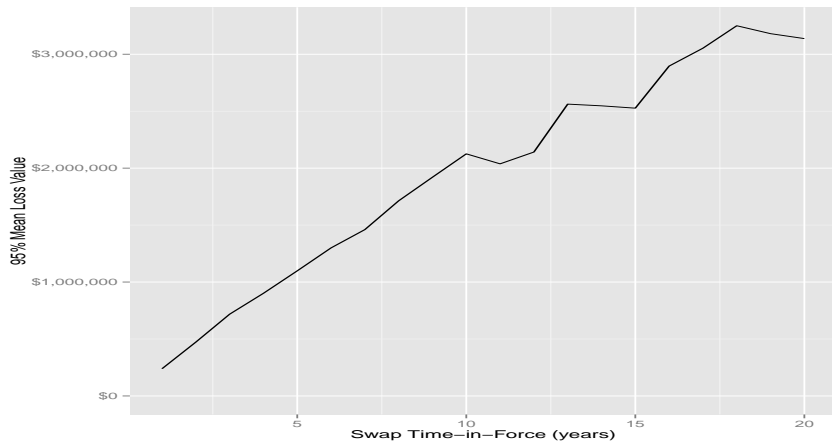
Pricing Tail Risk

Ensemble of 1,000 valuations of 10,000 iterations:



Time-in-Force Dependency

Scaling of 95% mean with time-in-force of swap:



Time-in-Force Dependency (Ensemble)

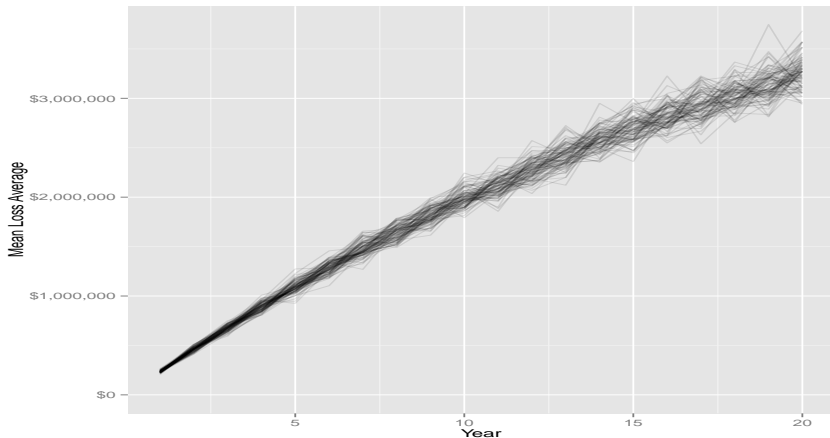
```
times.in.force <- 1:20;
n.sim          <- 1000;
n.ens         <- 10;

calculate.tif <- function(tif) {
  mortswap.value.sim <- calculate.mortality.swap.price(mortport.dt,
  lifetable           = lifetable.dt,
  hedge.apv.cashflows = TRUE,
  interest.rate       = 0.05,
  years.in.force      = tif,
  n.sim               = n.sim,
  return.all.data     = FALSE);

  interval <- calculate.quantile.averages(mortswap.value.sim, 0.95);
}

tif.ensemble <- sapply(times.in.force, function(iter.tif) {
  replicate(n.ens, calculate.tif(iter.tif))
});
```

Time-in-Force Dependency (Ensemble)



Summary

R package: `mcmortswap`

<https://bitbucket.org/appliedai/mcmortswap>

Email: michael.cooney@applied.ai

Slides available on bitbucket:

https://bitbucket.org/appliedai/r_in_insurance_2015

