Monte Carlo Valuation of Future Annuity Contracts

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Outline

Introduction

Propose a simulation based method to evaluate the distribution of future annuity values $\longrightarrow$ avoid nested simulations (very time-consuming).

Future annuity values are uncertain:

- Unknown future mortality (and interest/inflation) rates;
- Impact on liabilities for insurers/pension plans (Oppers et al., 2012);
- Impact on dependence between lifetimes (Alai et al., 2013, 2015 and Alai, 2019).

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State of the art

- Cairns (2011), Dowd et al. (2011) and Liu (2013): Taylor approximation-based approach $\rightarrow$ requires multiple simulation sets;
- Denuit (2008): comonotonic approximations.

Proposal

- Use the well-known LSMC method (Longstaff et al., 2001, Boyer et al., 2013, 2017);
- Flexible to accommodate any (Markov) mortality model;
- Extend to more general situations (see later).

The mathematical framework

Future value at T of an annuity contract with unitary benefits issued to an individual aged $x+T$ at the future date T as

$$a_{x+T}(T) = \sum_{i=1}^{\omega-x-T} B(T,T+i)\mathbb{E}\left[e^{-\sum_{h=0}^{i-1} m_{x+T+h;T+h}} \,\middle|\, \mathbf{z}_T\right], \tag{1}$$

where

- ω is the ultimate age;
- $B(T,T+i)$ is the i-th years discount factor prevailing at time $T>0$;
- $m_{x;t}$ is the central death rate at age x in year t;
- $\mathbf{z}_T$ is the state-vector of the relevant risk factors.

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Steps

1. Simulate $\mathbf{z}_t^{(j)} \to m_{x;t}^{(j)}, \qquad t = 1, \ldots, T$ and $j = 1, \ldots, n$;
2. For each outer scenario, projecting $\bar{n} \ll n$ inner paths of the risk factors (e.g., $\bar{n} = 1$);
3. Compute for each outer scenario the corresponding cash-flows generated along each inner trajectory, $\left\{A^{(j)}\right\}_{j=1,\ldots,n}$;
4. Regress

$$\left\{A^{(j)}\right\}_j \text{ on } \left\{\phi\left(\mathbf{z}_T^{(j)}\right)\right\}_j$$

   where $\phi = (\phi_1, \ldots, \phi_p)$ is a vector of basis functions;
5. Compute

$$\hat{a}_{x+T}^{(j)}(T) = \sum_{k=1}^{p} \hat{\beta}_k \phi_k \left(\mathbf{z}_T^{(j)}\right), \qquad j = 1, \ldots, n.$$

R FUNCTIONS

- Description
  - It implements the LSMC method for valuing future annuity contracts under stochastic mortality and interest rates framework.
- Usage
  - ```
    calculate.Annuity(mortRates, T, x, r = 0, close.table = TRUE,
      omega = 120, pred = NULL, basisFun = c("Monomials", "Hermite",
      "Laguerre", "Chebyshev"), ordPolyn = 1, standardize = TRUE)
    ```
- Arguments
  - `mortRates`: matrix/three-dimensional array, the future simulated mortality rates. It can be an object of class "`simStMoMo`".
  - `T`: integer value, the future time horizon.
  - `x`: integer value, the individual's age at the future date `T`.
  - `r`: constant/vector/matrix with future levels of interest rates.
  - ...
    ```

The class "sim.Annuity"

The class of the returned object is of type "`sim.Annuity`" which contains the following information:

- `annuity`: a vector containing the simulated future annuity values;
- `pred`: a matrix containing the predictors exploited in the regression;
- `basis`: a string indicating the type of basis functions;
- ....

Methods: `print`, `summary`, `mean`, `quantile`, `hist`, etc.

R code example

- `M7fit`: fitted Poisson M7 stochastic mortality model (`StMoMo` package) on
  - Italian male population $1965 - 2016$;
  - Ages $35 - 90$;
- `M7sim`: object of class "`simStMoMo`"
  - $n = 20000$ simulated trajectories;
- `CIRrates`: simulated future interest rates level (CIR process)
  - Parameters: $r_0 = 0.04$, $\alpha = 0.2$, $\bar{r} = 0.04$, and $\sigma_r = 0.1$;

Results


```
> Annuity_LSMC <- calculate.Annuity(mortRates = M7sim, T = 5, x = 65,
        ordPolyn = 1, r = CIRrates, pred = NULL, basisFun = "Monomials",
        close.table = FALSE)

> print(Annuity_LSMC)

Annuity values for an individual aged 65 at the future time horizon 5
 Contract Information
  Interest rate: stochastic
  Basis Functions: Monomials
  Number of Basis Functions: 5
  Number of Simulations: 20000
```

Results II

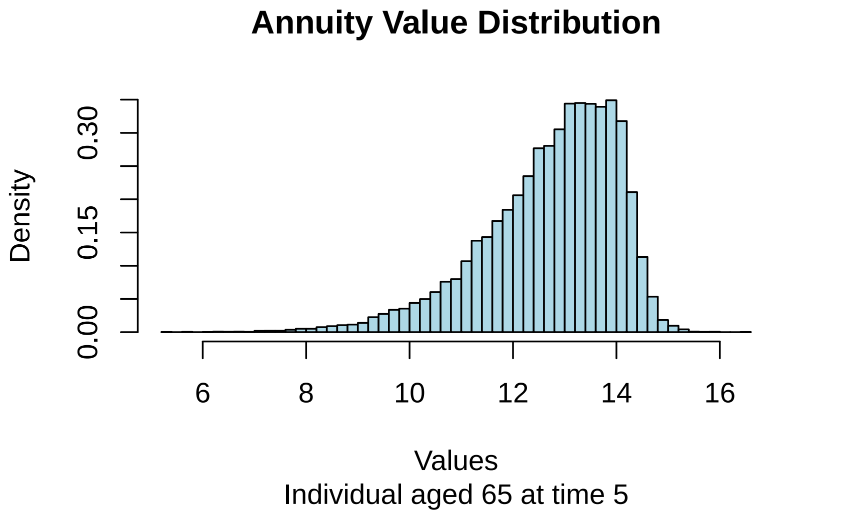
```
> summary(Annuity_LSMC)

   Min.  1st Qu. Median  Mean   3rd Qu.  Max.
  5.311  12.005  12.968 12.725 13.693 16.547

> quantile(Annuity_LSMC, p = c(0.005, 0.995))

     0.5%     99.5%
 8.211256 14.883423

> hist(Annuity_LSMC)
```


Annuity Value Distribution
Density
0.00
0.15
0.30
6
8
10
12
14
16
Values
Individual aged 65 at time 5

Thank you!

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