Risk Budgeting Portfolios from Simulations

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\textsuperscript{1}Joint work with Silvana Pesenti (UoT) and Bernardo Costa (UFRJ)
An investment concept used in industry, e.g., by pension funds, is **risk budgeting portfolios (RBP)**

- **RBP**: diversified portfolios on the level of the risk contribution of each asset
- **risk contribution**: the added risk of marginally increasing the portfolio's position in that asset

Our contribution: to use **stochastic optimization** to construct RBPs based on a generic **risk measure** and **simulations** from the returns.
The setup

- At time $t = 0$:
  - Assets prices: $\mathbf{p} = (p_1, \ldots, p_d)$
  - The investor holds: $a_i$ shares of asset $i$
  - Portfolio value: $\nu(\mathbf{a}) = \mathbf{a}^T \mathbf{p}$

- At time $t = 1$:
  - Assets prices: $\mathbf{P} = (P_1, \ldots, P_d)$
  - The investor holds: $a_i$ shares of asset $i$
  - Portfolio value: $\mathbf{V}(\mathbf{a}) = \mathbf{a}^T \mathbf{P}$

- Portfolio loss: $L(\mathbf{a}) = -[\mathbf{V}(\mathbf{a}) - \nu(\mathbf{a})] = \sum_{i=1}^{d} a_i L_i$
  - $i$-th asset loss: $L_i = -(P_i - p_i)$

- Initial endowment: $\nu_0$ (dollars) at time $t = 0$. 
Marginal Risk and Risk Contribution

- **Marginal risk** of asset $i$ (with coherent risk measure $\rho$)

  \[ M\mathcal{R}_i(a) = \frac{\partial}{\partial a_i} \rho(a^T L) \]

- **Risk contribution** of asset $i$

  \[ R\!C_i(a) = a_i M\!R_i(a) \].

- For a portfolio $a$ and a risk measure $\rho$, it holds

  \[ \rho(L(a)) = \sum_{i=1}^{d} a_i M\!R_i(a) = \sum_{i=1}^{d} R\!C_i(a) . \]
The Risk Budgeting Portfolio

- The investor has a risk appetite \( B^\dagger \) split as \( B^\dagger = \sum_{i=1}^{d} B_i \)
- For a risk budget \( B = (B_1, \ldots, B_d) \) the RB portfolio satisfies
  \[
  B^\dagger = \rho(L(a)) \quad \text{and} \quad B_i \mathcal{RC}_j(a) = B_j \mathcal{RC}_i(a)
  \]

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Proposition (RB portfolio)

*The RB portfolio’s weights are the solution to*

\[
a^* := \arg\min_a \rho(L(a)), \text{ subject to } \sum_{i=1}^{d} B_i \log(a_i) \geq 0,
\]

*where the \( a_i \) are implicitly constraint to be strictly positive, so that the log is well-defined.*
The Risk Budgeting Portfolio

- The investor has a **risk appetite** $B^{\dagger}$ split as $B^{\dagger} = \sum_{i=1}^{d} B_i$
- For a **risk budget** $B = (B_1, \ldots, B_d)$ the RB portfolio satisfies

  \[ B^{\dagger} = \rho(L(a)) \text{ and } B_i \mathcal{RC}_j(a) = B_j \mathcal{RC}_i(a) \]

- With **simulated** returns’ scenarios, we can estimate

  \[ \hat{\rho}(L(a)) \approx \rho(L(a)) \]

  and use the **Sample Average Approximation** method

**Proposition (SAA-RB portfolio)**

*The SAA-RB portfolio’s weights are the solution to*

\[
a^* \equiv \arg\min_a \hat{\rho}(L(a)), \text{ subject to } \sum_{i=1}^{d} B_i \log(a_i) \geq 0,
\]

*where the $a_i$ are implicitly constraint to be strictly positive, so that the log is well-defined.*
## Example: Multi-class RB portfolio

<table>
<thead>
<tr>
<th>Ticker</th>
<th>Name</th>
<th>Asset class</th>
<th>RB weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPY</td>
<td>SPDR S&amp;P 500 ETF</td>
<td>Equities</td>
<td>2</td>
</tr>
<tr>
<td>MDY</td>
<td>SPDR S&amp;P MidCap 400 ETF</td>
<td>Equities</td>
<td>1</td>
</tr>
<tr>
<td>VB</td>
<td>Vanguard Small-Cap ETF</td>
<td>Equities</td>
<td>1</td>
</tr>
<tr>
<td>GLD</td>
<td>SPDR Gold Shares</td>
<td>Commodities</td>
<td>2</td>
</tr>
<tr>
<td>DBC</td>
<td>PowerShares DB Comm. Tracking ETF</td>
<td>Commodities</td>
<td>1</td>
</tr>
<tr>
<td>DBA</td>
<td>PowerShares DB Agriculture ETF</td>
<td>Commodities</td>
<td>1</td>
</tr>
<tr>
<td>IEF</td>
<td>iShares 7–10 year Treasury Bond</td>
<td>Bonds</td>
<td>2</td>
</tr>
<tr>
<td>AGG</td>
<td>iShares Core U.S. Aggregate Bond</td>
<td>Bonds</td>
<td>1</td>
</tr>
<tr>
<td>TIP</td>
<td>iShares TIPS Bond</td>
<td>Bonds</td>
<td>1</td>
</tr>
</tbody>
</table>
Example: Multi-class RB portfolio
Example: Multi-class RB portfolio

Figure: SPY, MDY, VB, GLD, DBC, DBA, IEF, AGG, TIP
Example: Multi-class RB portfolio

<table>
<thead>
<tr>
<th></th>
<th>Vol.</th>
<th>Return</th>
<th>Sharpe</th>
<th>VaR&lt;sub&gt;5%&lt;/sub&gt;</th>
<th>Drawdown</th>
</tr>
</thead>
<tbody>
<tr>
<td>grp</td>
<td>5.72%</td>
<td>3.12%</td>
<td>0.545</td>
<td>-0.53%</td>
<td>-17.26%</td>
</tr>
<tr>
<td>gmv</td>
<td>5.96%</td>
<td>2.68%</td>
<td>0.450</td>
<td>-0.47%</td>
<td>-18.17%</td>
</tr>
<tr>
<td>ew</td>
<td>11.07%</td>
<td>5.11%</td>
<td>0.462</td>
<td>-1.00%</td>
<td>-30.58%</td>
</tr>
<tr>
<td>rp095</td>
<td>5.06%</td>
<td>2.71%</td>
<td>0.536</td>
<td>-0.47%</td>
<td>-15.86%</td>
</tr>
</tbody>
</table>

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