Metamodeling for Variable Annuity Valuation: What works and what does not

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Background and Literature

- Variable Annuities (VAs) are separate account life insurance contracts linked to a list of financial instruments with tax benefits
  - Equipped with various optional features
- Metamodeling for VA Portfolio Valuation: a very active research topic in actuarial science
  - Gan (2015), Gan & Lin (2017), Wu et al. (2018), Quan et al. (2021),...
  - *All* papers so far rely on synthetic datasets
- **Research Question: How well do metamodeling approaches work on real-world VA contracts?**
- Extract contract features and build a data set of VAs with GMABs
  - Implement a flexible MC simulation process for VA valuation
- Test Metamodeling with different sampling and learning components
  - Larger sample size ✓
  - Sophisticated learners ✓
  - Sampling methods ✗

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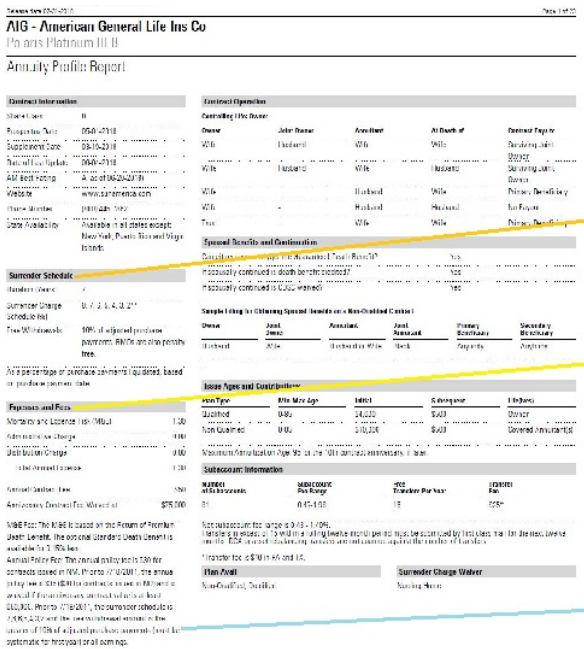
Real Variable Annuity Contract Data

- VA prospectus: Typically several hundred-page long documents with detailed description
  - Gan and Valdez (2017): “...extremely difficult, if not impossible, for researchers to obtain real datasets...”

- Our source: Morningstar Annuity Intelligence
  - 2,346 VA + GMAB combinations (22,623 in total for all benefit types)
  - From 1994 to 2017
  - Numerical values on fees and benefits
  - Textual description on features and conditions

Real Variable Annuity Contract Data

Morningstar Report Example


AIG - American General Life Ins Co

Polaris Platinum III B

Annuity Profile Report

Surrender Schedule	
Duration (Years)	7
Surrender Charge Schedule (%)	8, 7, 6, 5, 4, 3, 2**
Free Withdrawals	10% of adjusted purchase payments. RMDs are also penalty free.

As a percentage of purchase payments liquidated, based on purchase payment date.

Expenses and Fees	
Mortality and Expense Risk (M&E)	1.30
Administrative Charge	0.00
Distribution Charge	0.00
Total Annual Expense	1.30
Annual Contract Fee	$50
Anniversary Contract Fee Waived at	$75,000

M&E Fee: The M&E is based on the Return of Premium Death Benefit. The optional Standard Death Benefit is available for 0.15% less.
Annual Policy Fee: The annual policy fee is $30 for contracts issued in NM. Prior to 7/18/2011, the annual policy fee is $35 ($30 for contracts issued in ND) and is waived if the anniversary contract value is at least $50,000. Prior to 7/18/2011, the surrender schedule is 7,6,6,5,4,3,2 and the free withdrawal amount is the greater of 10% of adjusted purchase payments (must be systematic for first year) or all earnings.

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Real Variable Annuity Contract Data

Example of Feature Extraction

Feature Description	Textual Information	Data Extraction
Benefit Charge	0.750% assessed annually and calculated against the account value	fee_B_Base = AV; fee_B_Rate = 0.0075
Surrender Charge Schedule	7, 6, 4	Length = 3; Slope = -1.5; Max = 7; Min = 4
Impact of Withdrawal	Proportionate	IW = Proportionate
Step-up Option	On the 5th or subsequent anniversary the benefit base can be stepped-up to the current account value. Electing a step-up will restart the ten-year waiting period. Future step-ups are available five years after the last step-up.	SU_Base = reset; SU_Rate = 1; Initial = 5; Frequency = 1; Waiting = 10; Next = 5;
Specialities	(Benefit) Fee percentage is 0.55% prior to 11/1/2010, 0.40% prior to 3/2/2009 and 0.25% prior to 5/1/2008.	If ID < 11/1/2010: fee_B_Rate == 0.0055; If ID < 3/2/2009: fee_B_Rate == 0.0040; If ID < 5/1/2008: fee_B_Rate == 0.0025;

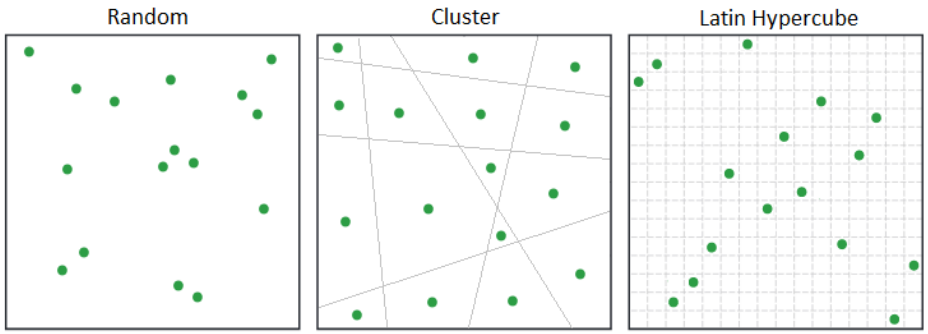
- 53 contract features for valuation and learning
- Black-Scholes framework with 648 scenarios for each contract

Metamodeling on VA valuation

Sample Selection and Statistical Learning

- **Select** $\Rightarrow$ **Calculate** $\Rightarrow$ **Learn** $\Rightarrow$ **Predict**

- Representative Sample Selection
  - Random Sampling
  - Clustering (k-means)
  - Latin Hypercube Sampling



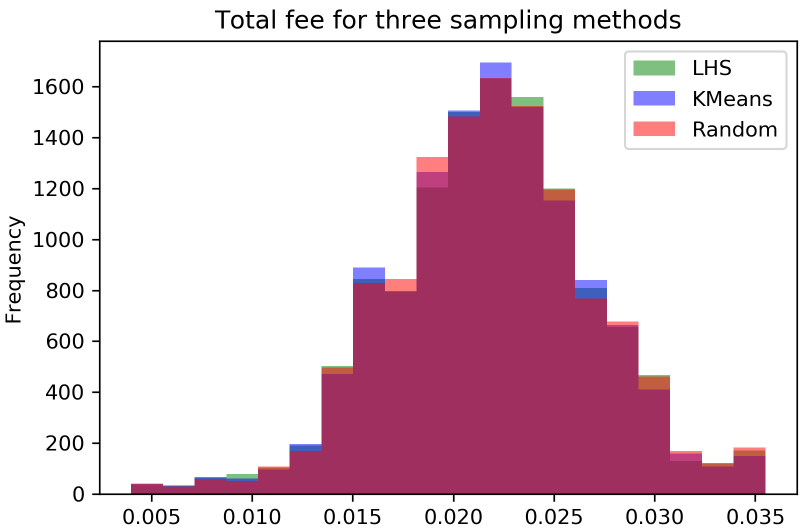
- Statistical Learning
  - Generalized Linear Model (baseline)
  - Tree-Based Models
  - Neural Network

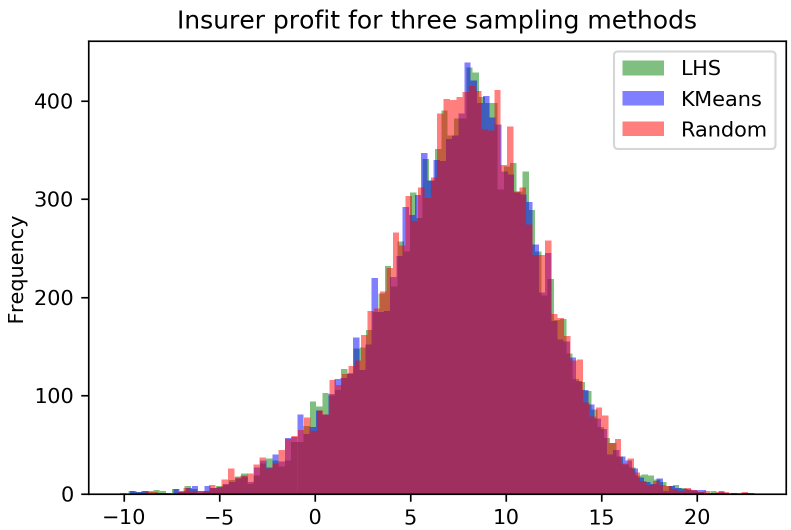
Metamodeling on VA valuation

Results on Sample Selection

- Similar distribution
- Time consuming vector quantization

	Random Sample			Latin Hypercube		Cluster Sampling	
Sample Size	1%	5%	20%	1%	5%	1%	5%
Sampling Time (h)	-	-	-	3.42	20.51	3.54	24.74





Metamodeling on VA valuation

Important Features (Standardized)

- GLM picks up dummies for feature categories
- Boosted Trees emphasize on fees

GLM		Boosted Trees	
Feature Name	Coefficient	Feature Name	Importance
IW Speciality	10.0609	BenefitFee	11.07%
StepUp Next	8.6718	SubAccountFee L	8.69%
StepUp Initial	-8.3399	VA Fee	6.98%
IW (min)	-5.0211	M&E Fee	6.16%
IW (dollar)	-3.4742	SubAccountFee U	6.06%
FreeWithdrawal Base (AV)	-3.182	WithdrawalStrategy 1	5.12%
FreeWithdrawal Base (BB)	-2.8338	SurranderCharge Slope	4.65%
BenefitFee Speciality	2.6186	Age	3.97%
WithdrawalStrategy 3	-2.4723	AnnuitizationAge	3.87%
BenefitFee Base (max)	2.4433	WithdrawalStrategy 3	3.75%

Metamodeling on VA valuation

Accuracy and Runtime of Metamodeling

		Random Sample			Latin Hypercube		Cluster Sampling	
	Sample Size	1%	5%	20%	1%	5%	1%	5%
GLM	Tuning Time (h)	-	-	-	-	-	-	-
	OOS RMSE	4.29	4.29	4.27	4.29	4.29	4.30	4.30
Boosted Trees	Tuning Time (h)	1.79	22.17	192.29	2.52	21.43	2.15	21.15
	OOS RMSE	3.77	3.15	3.04	3.78	3.20	3.77	3.21
Random Forest	Tuning Time (h)	0.08	0.43	2.70	0.08	0.44	0.13	0.56
	OOS RMSE	3.69	3.13	2.65	3.70	3.16	3.72	3.14
Neural Network	Tuning Time (h)	7.40	34.15	193.56	7.80	30.50	6.26	25.68
	OOS RMSE	4.00	3.53	3.45	4.02	3.53	4.13	3.55

- GLM isn't improving with more samples.

Metamodeling on VA valuation

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- Tuning time scales heavily with sample sizes
- RMSE decreases for about 20% with a 20$\times$ increase in sample size

Metamodeling on VA valuation

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- RMSE of \$2-\$3, with mean of the actual value around \$13 (s.d. of \$5)
- MAPE of 20%, PE on the portfolio level $< 0.1\%$

Thank you!