Hierarchical Compartmental Reserving Models

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Motivation

READY! FIRE! AIM!

vs.

AIM! READY! FIRE!
What are HCRM?

- Dynamical systems to describe the claims process
- Bayesian framework to capture uncertainties in data and expert knowledge
- Best implemented in probabilistic programming language such as Stan (e.g. via ‘brms’ in R) or PyMC3 to model, fit and simulate
When might you consider HCRM?

- Data is poor, but expert knowledge is rich
- Paid and outstanding claims to be modelled simultaneously
- Insight into the underwriting cycle desired
- Full distribution around cash flows needed
What are dynamical systems?

Dynamical systems are often used in physics, engineering and epidemiology to model a deterministic process.

Very flexible, but requires expert knowledge to model a process with differential equations and to parameterise.
Modelling diseases

- Susceptible
- Infectious
- Recovered

\[
\frac{dS}{dt} = -\beta IS \\
\frac{dI}{dt} = \beta IS - \gamma I \\
\frac{dR}{dt} = \gamma I
\]
Modelling diseases and claims are alike

- Susceptible
- Infectious
- Recovered
- Exposure
- Outstanding claims
- Paid claims

\[
\begin{align*}
\frac{dS}{dt} &= -\beta IS \\
\frac{dI}{dt} &= \beta IS - \gamma I \\
\frac{dR}{dt} &= \gamma I \\
\frac{dEX}{dt} &= -\beta \cdot EX \\
\frac{dOS}{dt} &= \beta \cdot RLR \cdot EX - \gamma \cdot OS \\
\frac{dPD}{dt} &= \gamma \cdot RRF \cdot OS
\end{align*}
\]
Model 1
Model 2

Exposure

Exposure is underwritten and earned

RLR

RLR $k_{er}$

claims going through a second stage, e.g., litigation

$OS_1$

Claims being processed directly

$OS_2$

Insurer pays claims

$RRF (k_{o1} + k_{o2})$

Paid

https://insurancecapitalmarkets.com
Model 2

Exposure
OS1

OS2
OS1 + OS2

Paid

Development period
Bayesian framework

Create a data generating model first:

- Consider process and parameter distributions

\[ y_j \sim D(f(t_j, \Theta), \Phi) \]

- Consider variance structure
- Consider hierarchical structure
  - Which parameters might have random effects, e.g. vary across accident years, development years, lines of business or entities?
Example

The diagrams illustrate the development of loss ratios (incr_lr) and cumulative loss ratios (cum_lr) over different years. The years shown are 1991 to 2000, with each year represented by different markers and colors.

- **incr_lr**: Shows the incremental loss ratios over the development years (Dev year) from 2 to 10, with peaks and fluctuations indicating changes in loss ratios over time.
- **cum_lr**: Shows the cumulative loss ratios, which increase steadily over the development years, indicating the accumulation of losses over time.

The data can be accessed at [https://insurancecapitalmarkets.com](https://insurancecapitalmarkets.com).
Hierarchical model candidate

Incremental paid loss ratio for AY $i$, dev year $j$

$$
\ell_{ij} \sim \text{Lognormal}(\eta(t_j; \theta, ELR_{i}), \sigma) \\
\eta(t; \theta, ELR_{i}) = \log(ELR_{i} \cdot (G(t_j; \theta) - G(t_{j-1}; \theta))) \\
= \log(ELR_{i}) + \log(G(t_j; \theta) - G(t_{j-1}; \theta)) \\
ELR_{i} \sim \text{Lognormal}(\log(ELR_{c}), \sigma_{[i]}) \\
ELR_{c} \sim \text{Lognormal}(\log(0.6), 0.1) \\
\sigma_{[i]} \sim \text{StudentT}(10, 0, 0.05)^+ \\
\theta \sim \text{Normal}(0.2, 0.02) \\
\sigma \sim \text{StudentT}(10, 0, 0.05)^+ 
$$
Simulated data vs observations

Posterior predictive model output against observations

Reserves: Aggregate future payments from latest observation
Distinguish between ELR and ULR

<table>
<thead>
<tr>
<th>AY</th>
<th>ELR (%)</th>
<th>Est. error</th>
<th>ULR (%)</th>
<th>Est. error</th>
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<td>1991</td>
<td>46.6</td>
<td>4.4</td>
<td>43.2</td>
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<td>5.4</td>
<td>58.7</td>
<td>2.2</td>
</tr>
<tr>
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<td>4.6</td>
<td>53.5</td>
<td>2.4</td>
</tr>
<tr>
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<td>4.8</td>
<td>50.2</td>
<td>2.9</td>
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<tr>
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<td>6.7</td>
<td>48.5</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Estimated ELR for each AY, e.g. underlying pricing loss ratio

Similar across AY

Projection from latest observation, i.e. required for reserving

Increasing across AY

https://insurancecapitalmarkets.com
Summary

- HCRM provide transparent framework for reserving
- Expert knowledge is part of the model design, not an add-on or afterthought
- Model can be useful to extract historical pricing information from claims data
- Paper has more details and case studies implemented in R and Stan using the ‘brms’ as an interface
Reference

Published article:


Online web version of the article:

https://compartmentalmodels.gitlab.io/researchpaper/index.html

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