ICMR.

Hierarchical Compartmental Reserving Models

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Motivation

READY! FIRE! AIM!

VS.

AIM! READY! FIRE!

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What are HCRM?

- Dynamical systems to describe the claims process
- Bayesian framework to capture uncertainties in data and expert knowledge
- Best implemented in probabilistic programming language such as Stan (e.g. via ‘brms’ in R) or PyMC3 to model, fit and simulate

When might you consider HCRM?

- Data is poor, but expert knowledge is rich
- Paid and outstanding claims to be modelled simultaneously
- Insight into the underwriting cycle desired
- Full distribution around cash flows needed

What are dynamical systems?

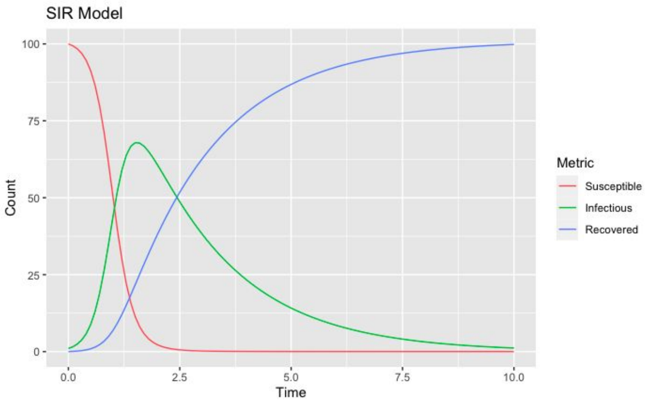
Dynamical systems are often used in physics, engineering and epidemiology to model a deterministic process

Very flexible, but requires expert knowledge to model a process with differential equations and to parameterise

Modelling diseases

- Susceptible
- Infectious
- Recovered

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$



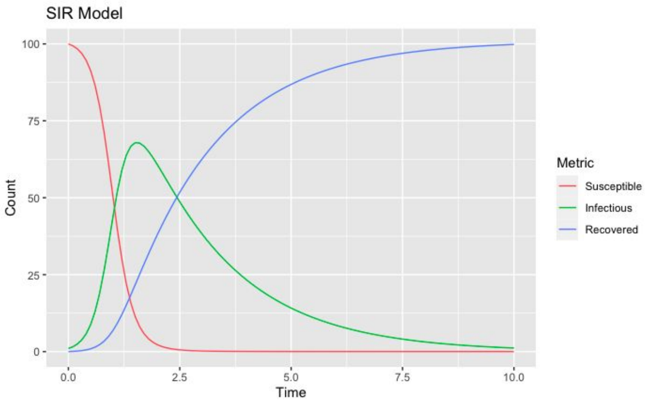
Modelling diseases and claims are alike

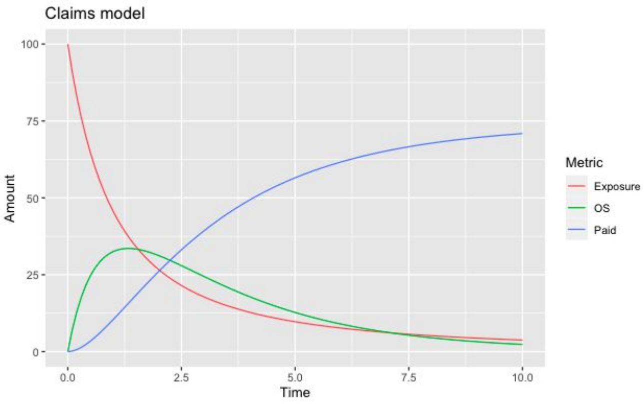
- Susceptible
- Infectious
- Recovered

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

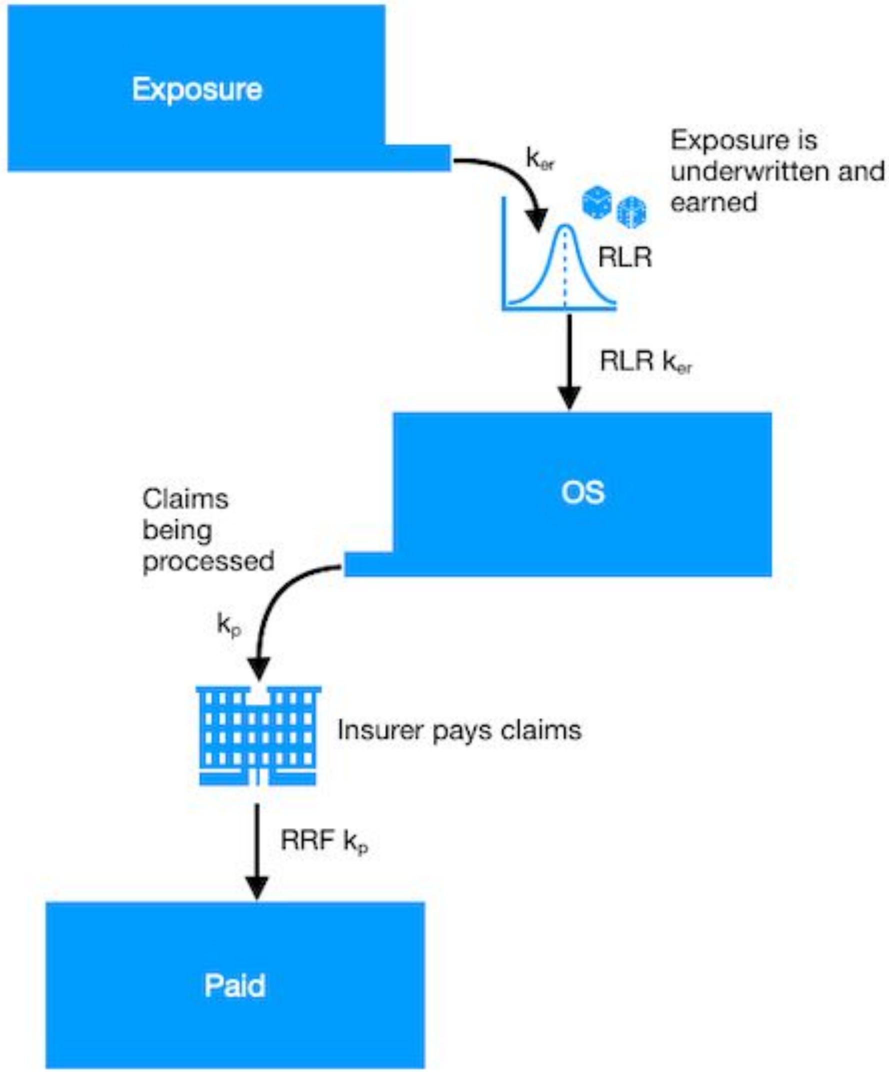
- Exposure
- Outstanding claims
- Paid claims

$$\frac{dEX}{dt} = -\beta \cdot EX$$
$$\frac{dOS}{dt} = \beta \cdot RLR \cdot EX - \gamma \cdot OS$$
$$\frac{dPD}{dt} = \gamma \cdot RRF \cdot OS$$



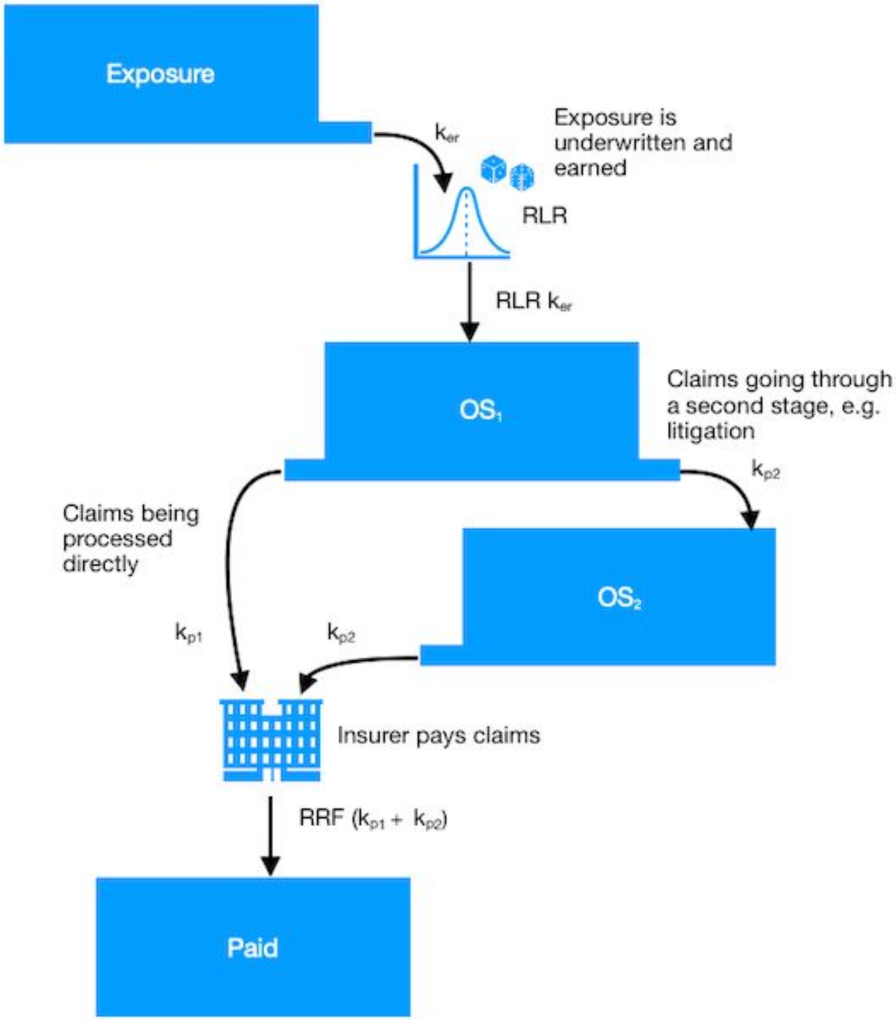


Model 1

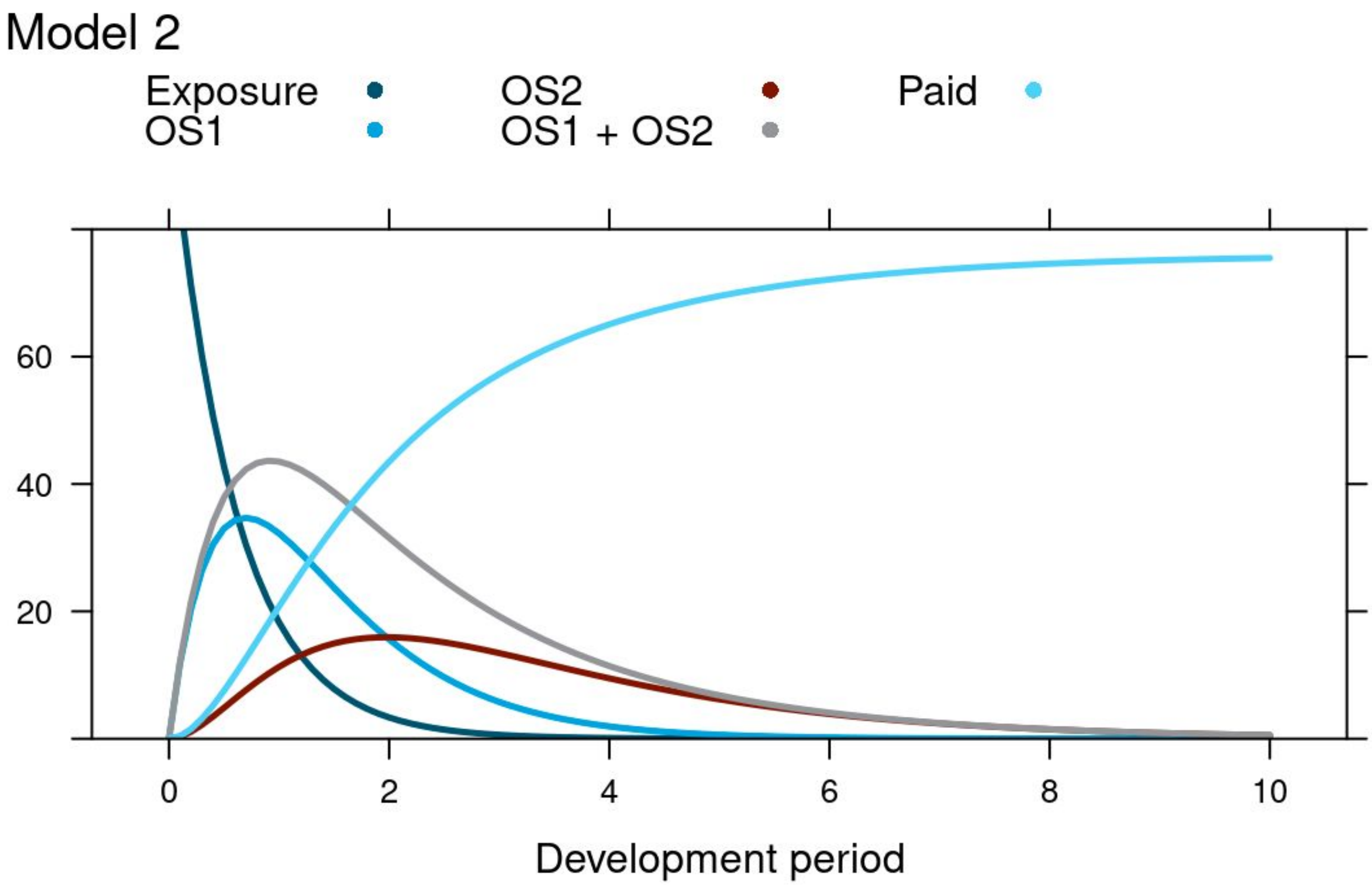

Exposure
k_{er}
Exposure is underwritten and earned
RLR
RLR k_{er}
OS
Claims being processed
k_p
Insurer pays claims
RRF k_p
Paid

ICMR.

Model 2



Exposure
k_{er}
Exposure is underwritten and earned
RLR
RLR k_{er}
OS₁
Claims going through a second stage, e.g. litigation
k_{p2}
Claims being processed directly
OS₂
k_{p1}
k_{p2}
Insurer pays claims
RRF (k_{p1} + k_{p2})
Paid


Model 2
Exposure
OS1
OS2
OS1 + OS2
Paid
60
40
20
0
2
4
6
8
10
Development period

Bayesian framework

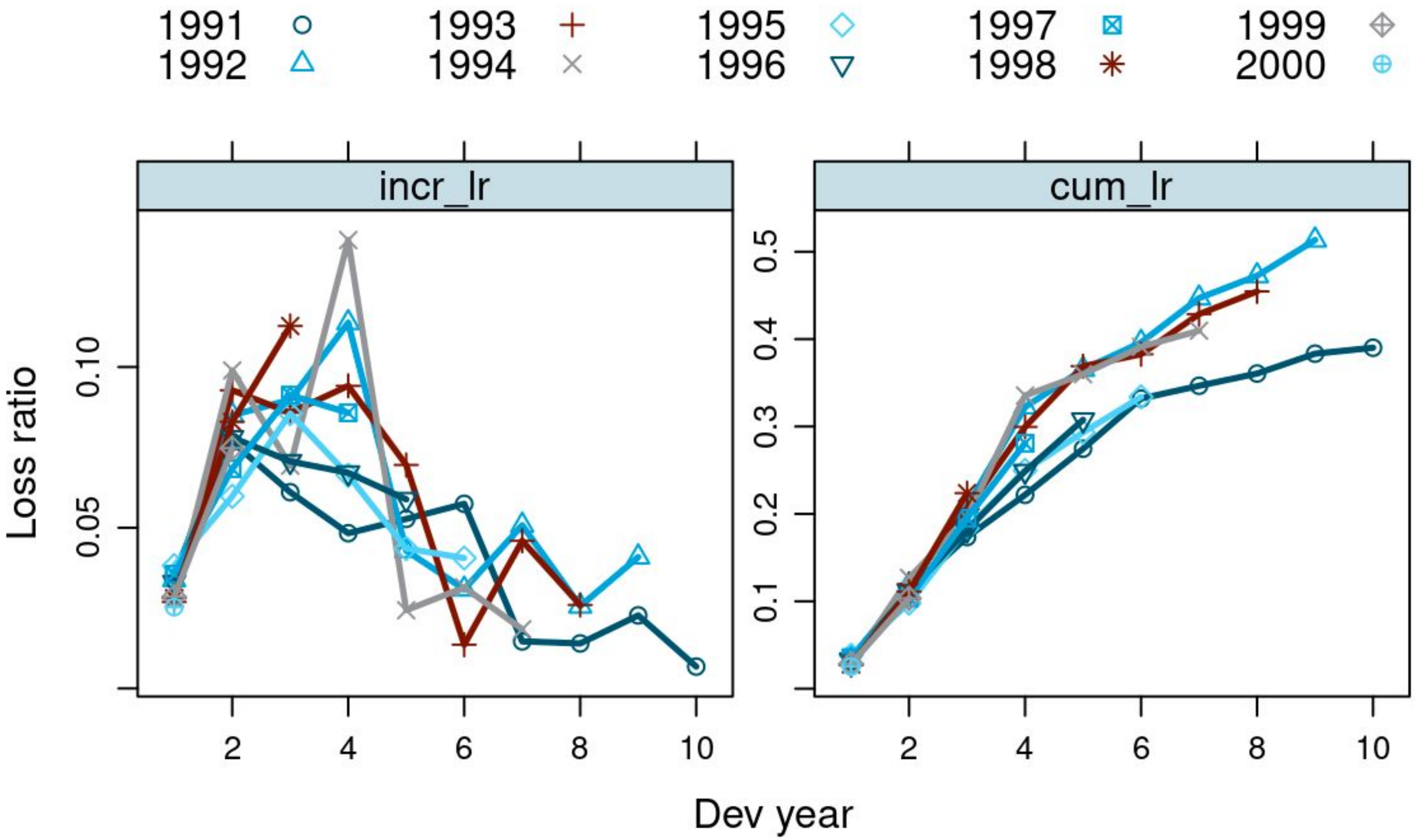
Create a data generating model first:

- Consider process and parameter distributions

$$y_j \sim D(f(t_j, \Theta), \Phi)$$

- Consider variance structure
- Consider hierarchical structure
  - Which parameters might have random effects, e.g. vary across accident years, development years, lines of business or entities?

Example

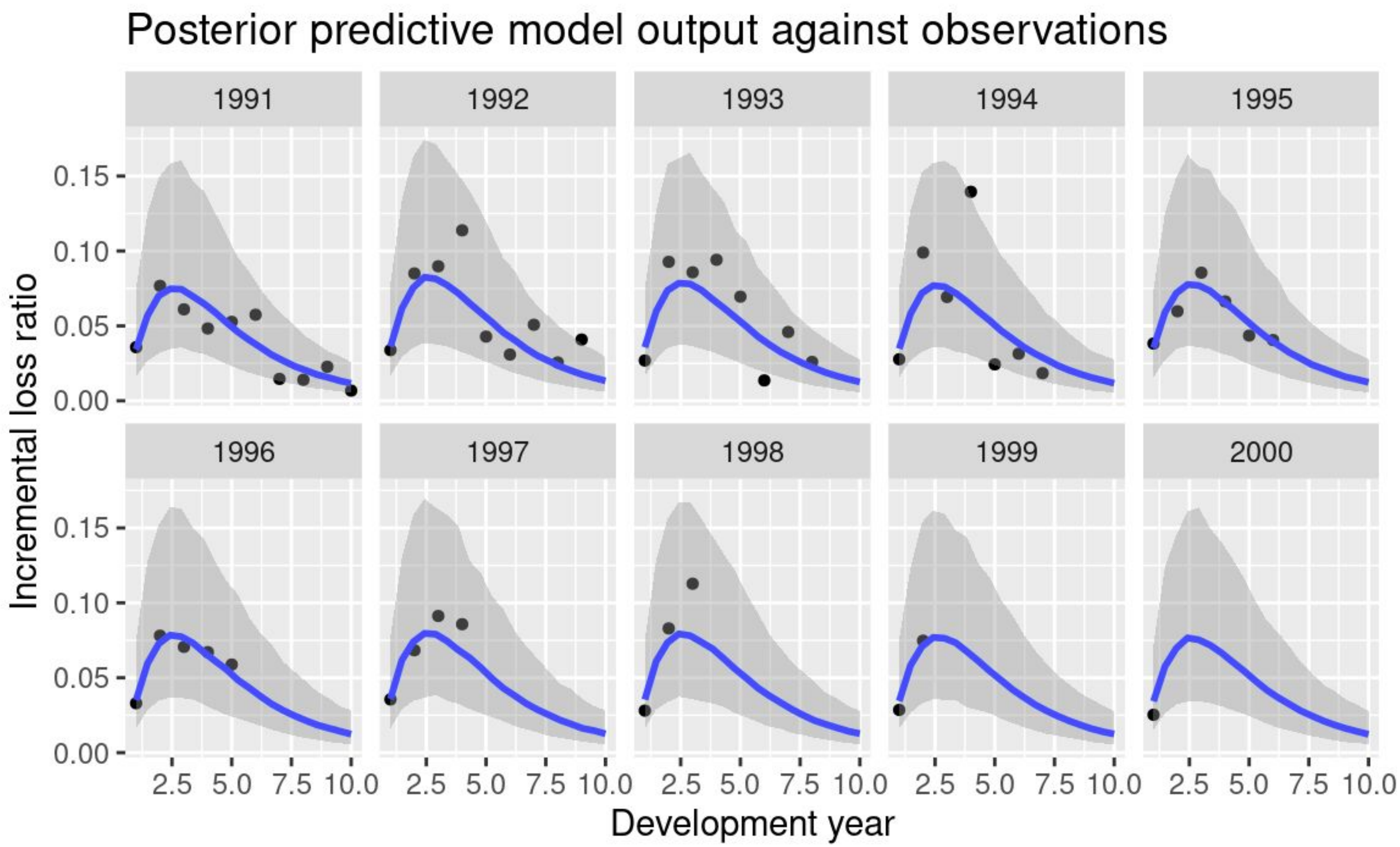

1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
incr_lr
cum_lr
Loss ratio
0.05
0.10
0.1
0.2
0.3
0.4
0.5
2
4
6
8
10
Dev year

Hierarchical model candidate

Incremental paid loss ratio for AY i, dev year j

$$
\begin{aligned}
\ell_{ij} &\sim \text{Lognormal}(\eta(t_j;\theta, ELR_{[i]}), \sigma) \\
\eta(t;\theta, ELR_{[i]}) &= \log(ELR_{[i]} \cdot (G(t_j;\theta) - G(t_{j-1};\theta)) \\
&= \log\left(ELR_{[i]}\right) + \log\left(G(t_j;\theta) - G(t_{j-1};\theta)\right) \\
ELR_{[i]} &\sim \text{Lognormal}(\log(ELR_c), \sigma_{[i]}) \\
ELR_c &\sim \text{Lognormal}(\log(0.6), 0.1) \\
\sigma_{[i]} &\sim \text{StudentT}(10, 0, 0.05)^{+} \\
\theta &\sim \text{Normal}(0.2, 0.02) \\
\sigma &\sim \text{StudentT}(10, 0, 0.05)^{+}
\end{aligned}
$$

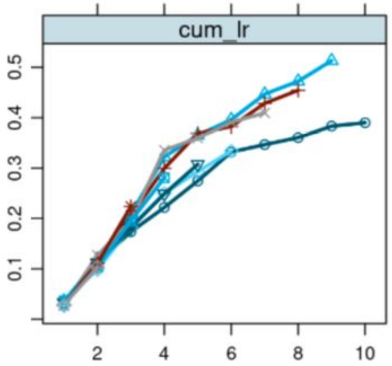
Simulated data vs observations



Reserves: Aggregate future payments from latest observation

Distinguish between ELR and ULR

AY	ELR (%)	Est. error	ULR (%)	Est. error
1991	46.6	4.4	43.2	1.4
1992	52.7	5.4	58.7	2.2
1993	49.7	4.6	53.5	2.4
1994	47.4	4.8	50.2	2.9
1995	49.0	4.9	47.0	3.8
1996	49.5	5.4	49.1	4.7
1997	50.5	6.1	52.2	5.7
1998	50.4	6.0	53.8	6.7
1999	48.7	6.2	49.1	7.7
2000	48.3	6.7	48.5	8.6



Estimated ELR for each AY, e.g. underlying pricing loss ratio

Similar across AY

Projection from latest observation, ie required for reserving

Increasing across AY

Summary

- HCRM provide transparent framework for reserving
- Expert knowledge is part of the model design, not an add-on or afterthought
- Model can be useful to extract historical pricing information from claims data
- Paper has more details and case studies implemented in R and Stan using the 'brms' as an interface

Reference

Published article:

Gesmann, M., and Morris, J. "Hierarchical Compartmental Reserving Models." Casualty Actuarial Society, CAS Research Papers, 19 Aug. 2020, https://www.casact.org/sites/default/files/2021-02/compartmental-reserving-models-gesmannmorris0820.pdf

Online web version of the article:

https://compartmentalmodels.gitlab.io/researchpaper/index.html

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