Relativities in the Over-dispersed Poisson Bootstrap Claims Reserves

Queensley C. Chukwudum (PhD)

Department of Insurance and Risk Management, University of Uyo, Uyo, Akwa Ibom State, Nigeria.

Paper presented at the:
3^{rd} Insurance Data Science Conference
The Business School (formerly Cass), City, University of London
(On-line)16 - 18 June 2021.

Outline

- Why is this investigation needed?
- Methods used.
- Data, Analysis, Results and Discussion.
- Conclusion.
- References.

Why is this investigation needed?
Background

- Claims can take a several years to settle, nevertheless they significantly affect the pricing of insurance policies.
- $\Rightarrow$ Assessing future claims payments is of utmost importance to an insurance company. In particular, stochastic reserving models have the ability to accomplish this task with the added advantage of being able to quantify the uncertainty inherent in the reserving process.
- But they do underestimate the variability of the outcomes of loss reserves due to the correlations. (Leong et al., 2014)

Why is this investigation needed?
Literature

- Some models allow for modelling of correlations in order to increase accuracy for example, generalized Mack chainLadder; general multivariate CL model; Bayesian non-linear hierarchical model; Tweedie technique; Bayesian MCMC estimation method; gamma moving average achieved using Poisson latent variables .

- Very few researchers have studied the relativities present in the structure of the ODP bootstrap models.
  - Leong et al. (2014) back-tested the ODP bootstrap of the paid chain-ladder model data from hundreds of U.S. companies.Their results indicated an underestimation of reserving risk.

- Moreover, studies are lacking in this respect as it relates to its application with African datasets.

Why is this investigation needed?
Objective

- **Objective:** Structural relationship between the ODP bootstrap case reserves (CR) and the technical provisions, particularly, the current year (CTY) and the immediate next year (NTY) .

  Specifically,

  - Correlations between CTY and NTY are also examined in order to understand the possible dependencies within the the run-off triangles.
  - Can we roughly predict CR given only CTY and NTY?
  - Is the interaction between CTY and NTY important in the prediction of CR?

Methods used

The basic ODP model / GLM framework

- The quasi-Poisson developed by Wedderburn (1974) accounts for the solution to the over-dispersion problem when using the Poisson distribution.
- The ODP bootstrap methodology follows a two-stage procedure:
  First stage:
  - Apply the quasi-Poisson model to the claims triangle in order to forecast future payments.
  - Draw multiple times, the scaled-Pearson residuals (represented as $\frac{x-\mu}{\sqrt{\phi\mu}}$), with replacement to generate bootstrapped (pseudo) triangles.
  - Use the bootstrapped triangles to estimate the error in the parameter as well as forecast future payments of claims.

  Second stage:
  - Obtaining a predictive distribution by simulating the process error using the bootstrap value as the mean and the quasi-Poisson model as the process distribution.

Methods used

The basic ODP model / GLM framework

- A generalized linear (GLM) is composed of three components: i) random component, ii) systematic component, iii) link function.
- Based on the Shapland's (2016) notation, The Poisson model can be cast into or formulated as a GLM as follows:

$$E[q(w,d)] = M_{w,d}$$

$$Var[q(w,d)] = \phi E[q]$$

$$\ln[M_{w,d}] = \eta_{w,d}$$

$$\eta_{w,d} = c + \alpha_w + \beta_d$$

Where $w = 1, 2, 3, ..., n; d = 1, 2, 3, ..., n; \alpha_1 = \beta_1 = 0$

The values of z indicates the distribution to be used ($z = 0$ indicates Normal, $z = 1$ indicates Poisson).

ϕ is the scale parameter

q represents the incremental data modeled as Poisson random variables having a log link function

$\eta_{w,d}$ is the link function needed to linearize the multiplicative model.

GLM as a supervised machine learning tool:

- This interaction between two predictors can generally be represented as:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + error\ term$$

  Where: β_0 represents the overall average response, β_1 and β_2 represent the average rate of change due to x_1 and x_2, β_3 represents the average rate of chage due to the interacting variables x_1 and x_2.

- In this paper the β_i parameters will be estimated using the GLM where Y is taken as the mean CR and x_1 and x_2 are CTY and NTY respectively.

Data, Analysis, Results and Discussion

Data

- Nigeria: The cumulative inflation-adjusted paid claims development table for all claims (excluding large claims) was extracted from the 2018 Allianz (Nigeria) annual report. The accident years run from 2007 (the base year) to 2018 with 12 development lags.
- US: data is gotten from the Casualty Actuarial Science (CAS) loss reserve database. That is, schedule P loss reserve triangles from the NAIC annual statement. Incurred losses (net of reinsurance) corresponds to claims of 10 accident years - 1988 to 1997 having 10 development lags.
- UK motor dataset is extracted directly from the ChainLadder R package. It consists of 7 accident years (2007-2013) having 7 development lags.

Analysis: Procedure

- To examine the relativities present, the incremental triangles are used.
- The bootstrap chainladder was run having ODP model as the process distribution with sample size n=1000.
- The bootstrapped triangles are then used to predict the expected value of future payments (CR).
    - The payment distribution over the immediate next year (NTY) is obtained .
    - The payment distribution for the current year, CTY, is arrived at using the bootstrapped simulated claims.
    - This gives 1000 sets each of NTY and CTY

Scatterplots

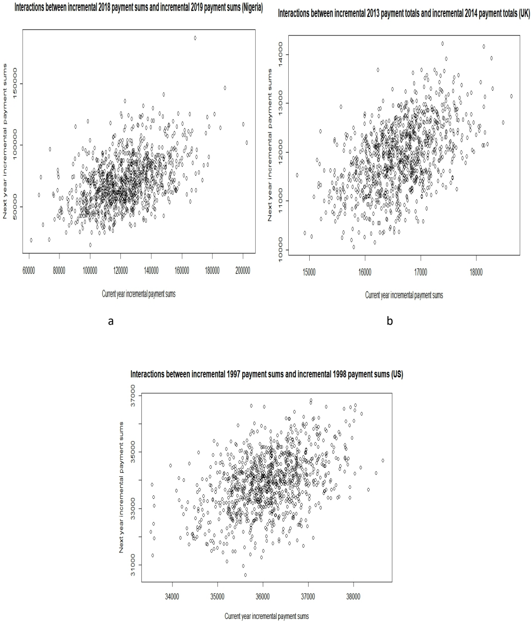

Figure 1: Scatterplots of CR totals with respect to both NTY and CTY. Nigeria (a),UK (b) and US (c).

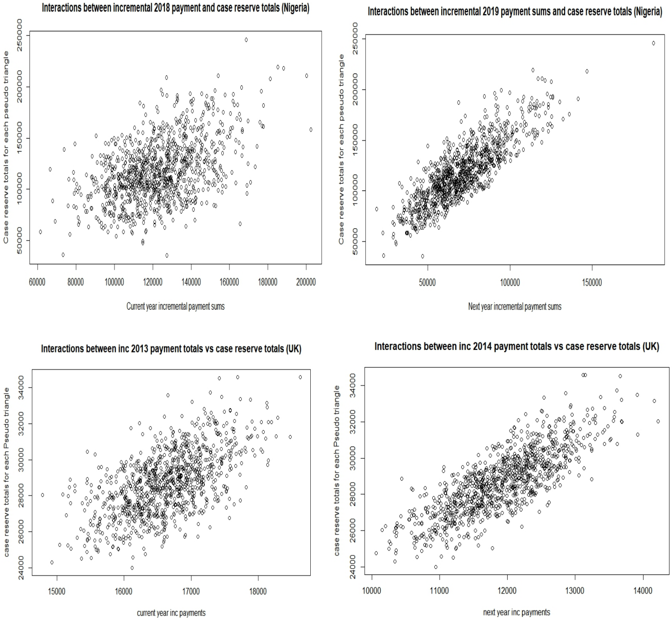

Figure 2: Plots of CR and CTY (LHS) and between CR and NTY (RHS). Nigeria (top) and UK (bottom).

Analysis: Correlation; Supervised learning

Table 1: Kendall and Spearman correlation results

		CR vs CTY	CR vs NTY	CTY vs NTY
(NG)	Kendall	0.296	0.626	0.320
	Spearman	0.429	0.817	0.451
(UK)	Kendall	0.405	0.592	0.334
	Spearman	0.575	0.790	0.487
(US)	Kendall	0.443	0.475	0.283
	Spearman	0.621	0.656	0.410

Training and testing the GLM models

- The dataset is then partitioned into training and test sets in the ratio of 7:3.
- Two GLM models are trained using the train data set and predictions are run using the test set - model 1 assumes all predictor variables are independent; model 2 accounts for the potential interaction between CTY and NTY.

Results:Case reserves and error estimates

Table 2: Estimated case reserves for model 1 and model 2

	Original	Model 1 (no interaction)	Model 2 (with interaction)
Nigeria	118,036	118,845	118,831
UK	28,709	28,687	28,686.59
US	181,496	181,095	181,108

Table 3: Estimated AIC and RMSE of prediction for model 1 and model 2

	Model 1 (no interaction)		Model 2 (with interaction)	
	AIC	RMSE	AIC	RMSE
Nigeria	1612545	18035.02	1612411	17930.05
UK	31458	1007.21	31457	1007.26
US	65379	4180.68	65294	4172.91

Remark: Predictive power of the GLM is slightly improved interactionare accounted for. (UK case remains unchanged even when n=10000).

Conclusion: Main take-a-ways

- **Correlations within triangles in ODP bootstrap model:** Yes. CR with respect to NTY payments exhibit higher positive correlations.
- **Can we roughly predict CR given only CTY and NTY?:** Yes!
- **Is the interaction between CTY and NTY important in the prediction of CR?:** Most probably, yes.
- **Future pathway:** How the interacting variables (CTY and NTY) are evolving and how the case reserves respond to changing/weighted combinations of these independent variables.

References

- Leong, J., Wang, S. and Chen, H. (2014). Back-Testing the ODP Bootstrap of the Paid Chain-Ladder Model with Actual Historical Claims Data. Variance 8(2), 182-202.
- Wedderburn, R. W. M. (1974). Quasi-likelihood Functions, Generalized Linear Models, and the Gauss-Newton Method. Biometrika 61(3), 439–447.
- Shapland, M.R. (2016). Using the ODP Bootstrap Model: A Practitioner's Guide. CAS Monograph Series Vol 4, CAS Actuarial Society.

THANK YOU FOR LISTENING.