From Chain Ladder to Probabilistic Neural Networks for Claims Reserving

https://institute-and-faculty-of-actuaries.github.io/mlr-blog/post/research/chain_ladder_to_individual_mdn/

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What is the potential with neural networks?

Advantages:

- Residual networks generalize GLMs
- Used in state-of-the-art models for e.g. image, text, audio transcriptions
- Transformers quite powerful for sequence data generally
- Entity embeddings can effectively model categorical variables
- Output probability distributions with mixture density

Issues to consider:

- In time series, simpler models often perform as well as neural networks (“Are Transformers Effective for Time Series Forecasting?” A. Zeng et al)
- With tabular-only data, gradient boosted decision trees are often easy to calibrate to a good result.
- Random initialization may lead to variance in model predictions
- Complexity vs simpler models

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About the author

- Member of Machine Learning in Reserving Working Party (with IFoA)
- Head of Finance at nib Travel
  - Experience in pricing & analytics
- Convenor for the Young Data Analytics Working Group (with Actuaries Institute in Australia)
  - Newsletter, podcast, articles, events
  - Check out our "Actuaries' Analytical Cookbook"! https://actuariesinstitute.github.io/cookbook/docs/index.html

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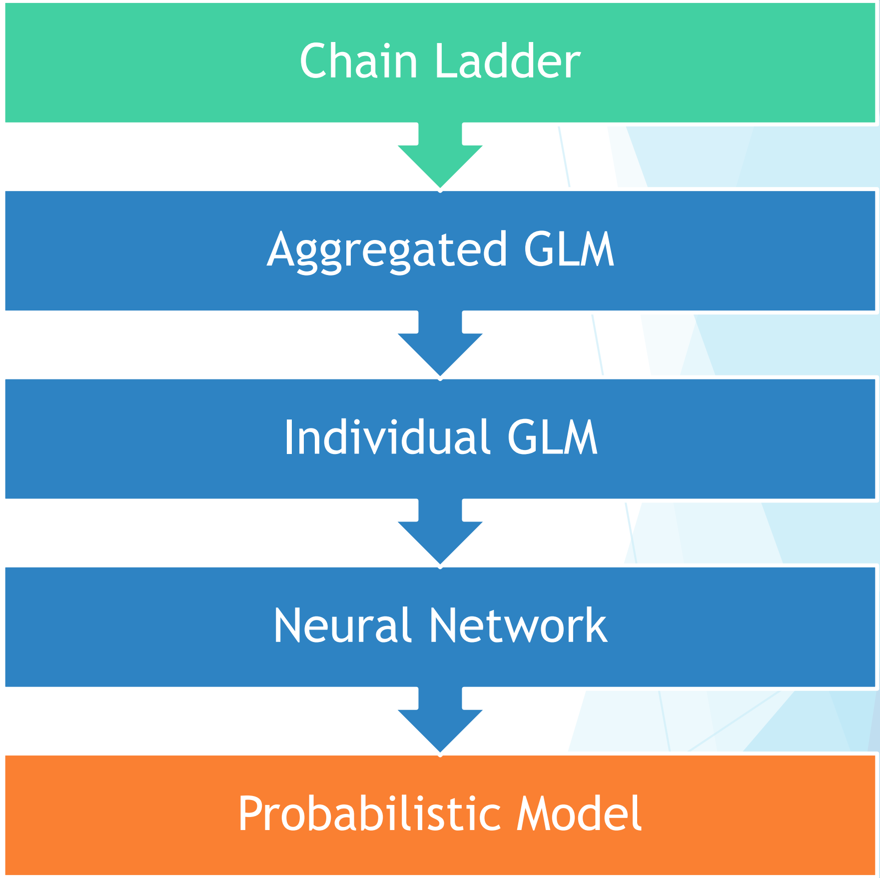
The journey we took:

- Start with a chain ladder model
- Make incremental changes
- Working model at each step
- Finish with probabilistic neural network model
- Simple simulated dataset
- Code available: https://institute-and-faculty-of-actuaries.github.io/mlr-blog/post/research/chain_ladder_to_individual_mdn/

Share some insights from the journey…



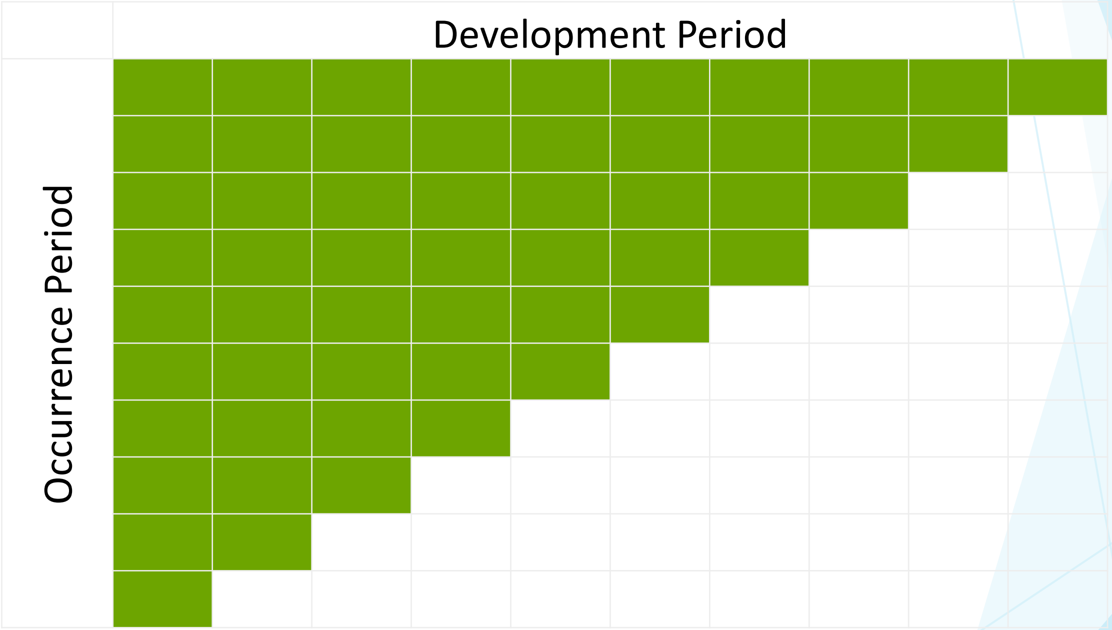
Key Observations

From this journey

Chain ladder is a GLM

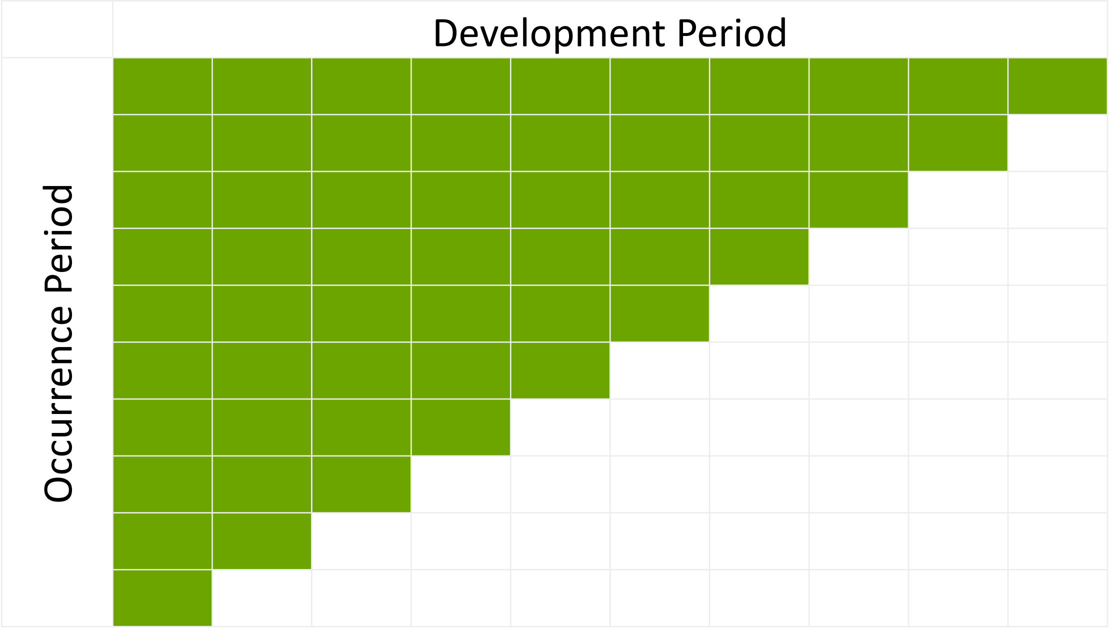
There is a GLM form of chain ladder:

- Log link, over-dispersed Poisson
- *Incremental Payments ~ Occurrence Period + Development Period*
- Occurrence Period and Development Period are one-hot encoded (1-0 flags for occurrence and development period = n, n=1...N)
- See https://institute-and-faculty-of-actuaries.github.io/mlr-blog/post/foundations/python-glms/

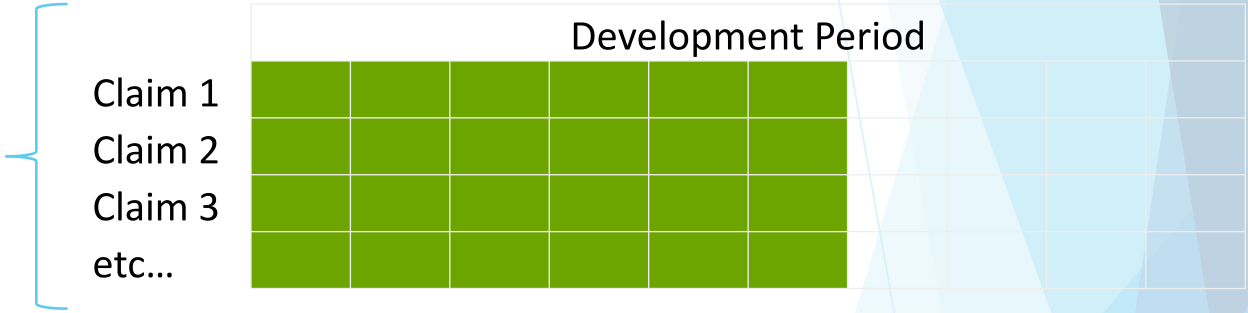


CL inspires an individual data format:

Chain ladder GLM: record per occurrence period x development period



“Zoom in”



Instead, use one record per:
claim number x development period

→ Per claim projection (IBNER)

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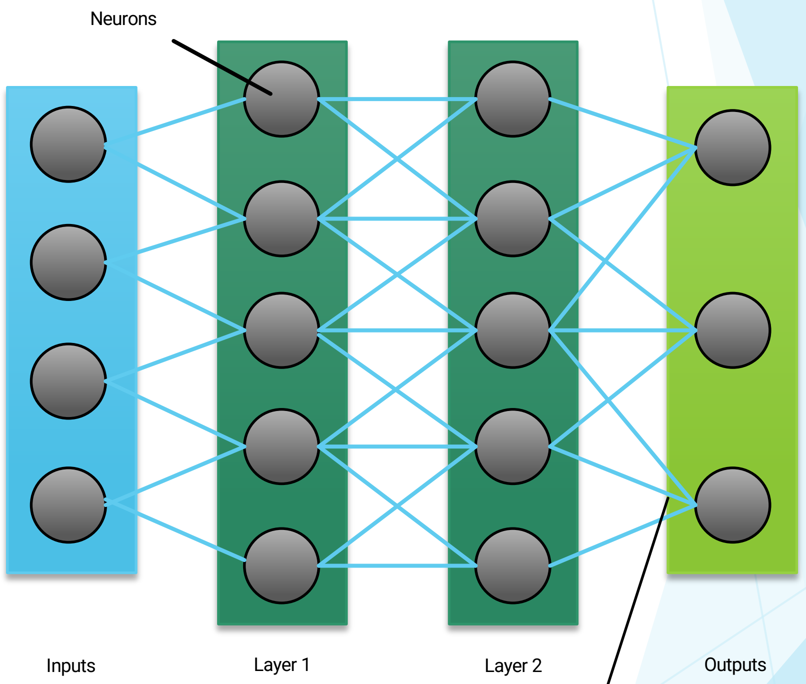
A linear model is a neural network

(With no hidden layers)

- A feedforward neural network is a type of neural network where information flows in one direction, from the input layer through one or more hidden layers to the output layer.
- Each layer in a feedforward neural network consists of a set of nodes, or neurons, that perform a **linear transformation of their inputs** followed by a non-linear activation function.
- The linear transformation performed by each neuron is similar to that of a linear model: the inputs are multiplied by a set of weights, and a bias term is added to the result.
- The activation function applied to the output of each neuron introduces non-linearity into the model, allowing it to learn complex relationships between the input and output variables.
- The outputs are a linear transform of the final hidden layer.
- **Consequently, a feedforward network can be considered a linear model of features, being the final hidden layer.**



The final step is linear!

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A linear model is a neural network

With no hidden layers

```python
# One hidden layer (simplified example)
class FeedForwardNet(nn.Module):
    def __init__(self, n_input, n_hidden, n_output):
        super(FeedForwardNet, self).__init__()
        self.hidden = nn.Linear(n_input, n_hidden)
        self.linear = nn.Linear(n_hidden, n_output)

    def forward(self, x):
        x = self.hidden(x)
        x = F.relu(x)
        x = self.linear(x)
        return x
```

```python
# No hidden layers = LM
class LinearModel(nn.Module):
    def __init__(self, n_input, n_output):
        super(LinearModel, self).__init__()

        self.linear = nn.Linear(n_input, n_output)

    def forward(self, x):


        x = self.linear(x)
        return x
```

Gradient descent methods can be used to fit instead of iterated reweighted least squares - minimize normal loss to maximise likelihood

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A GLM is also a neural network

```python
# Linear model
class LinearModel(nn.Module):
    def __init__(self, n_input, n_output):
        super(LinearModel, self).__init__()

        self.linear = nn.Linear(n_input, n_output)

    def forward(self, x):

        x = self.linear(x)
        return x
```

```python
# Add an exponential activation at the end
class LogLinkGLM(nn.Module):
    def __init__(self, n_input, n_output):
        super(LogLinkGLM, self).__init__()

        self.linear = nn.Linear(n_input, n_output)

    def forward(self, x):

        x = self.linear(x)
        return torch.exp(x)  # log(Y) = XB -> Y = exp(XB)
```

Add exponential activation to convert from a linear model to a GLM with log link. Poisson loss function can be minimized to fit the GLM.

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Neural networks can be "GLM+"

```
# GLM
class LogLinkGLM(nn.Module):
    def __init__(self, n_input, n_output):
        super(LogLinkGLM, self).__init__()



        self.linear = nn.Linear(n_input, n_output)


    def forward(self, x):

        x = self.linear(x)
        return torch.exp(x)  # log(Y) = XB -> Y = exp(XB)
```

```
# "Log Link" ResNet
class LogLinkResNet(nn.Module):
    def __init__(self, n_input, n_hidden, n_output):
        super(LogLinkResNet, self).__init__()
        self.hidden = nn.Linear(n_input, n_hidden)
        self.neural = nn.Linear(n_hidden, n_output)
        self.linear = nn.Linear(n_input, n_output)

    def forward(self, x):
        h = F.elu(self.hidden(x))
        x = self.linear(x) + self.neural(h)
        return torch.exp(x)  # log(Y) = XB -> Y = exp(XB)
```

A residual network is similar to a linear model with additional non-linear deep learning features. By including the exponential transformation at the final step, results become similar to a log-link GLM.

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Neural networks are flexible

- We test out our "SplineNet" design:
  - Split inputs into individual features
  - For each feature, fit a hidden layer on just that feature as a one-way "spline"
  - Fit an interaction hidden layer on all inputs as per a residual network but
  - Hide the interaction layer behind a "gate" weight, which is initialized in an "off" state

```
# The forward function defines how you get y from X.
def forward(self, x):
    # Apply one-ways
    chunks = torch.split(x, [1 for i in range(0, self.n_input)],
    dim=1)
    splines = torch.cat([self.oneways[i](chunks[i]) for i in
    range(0, self.n_input)], dim=1)

    # Sigmoid gate
    interact_gate = torch.sigmoid(self.interactions)

    splines_out = self.oneway_linear(F.elu(splines)) * (1 -
    interact_gate)
    interact_out =
    self.linear(F.elu(self.hidden(self.dropout(x)))) *
    (interact_gate)
    # Add ResNet style
    return self.inverse_of_link_fn(splines_out + interact_out)
```


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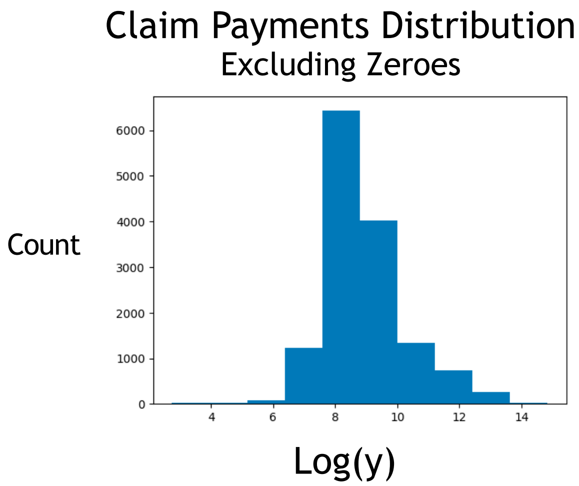
Lognormal Mixture Density Network

- **Capture variability:** Output variable modelled as the weighted sum of log-normal distributions.



- α is weight of each distribution
- μ and σ is lognormal's μ and σ
- Mean is $\sum_{k=1}^{n} a_k \cdot e^{(\mu_k + \frac{\sigma_k^2}{2})}$
- Some tricks to ensure numerical stability (details in notebook)

```
SMALL = 1e-7

def log_mdn_loss_fn(y_dists, y):
    y = torch.log(y + SMALL) # log(y) ~ Normal
    alpha, mu, sigma = y_dists
    m = torch.distributions.Normal(loc=mu, scale=sigma) # Normal
    loss = -torch.logsumexp(m.log_prob(y) + torch.log(alpha + 1e-15), dim=-1)
    return torch.mean(loss) # Average over dataset
```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Tips and tricks for neural networks for claims data

- Initialisation strategy: Bias: Set to mean(log(y)) to converge faster, Weights: Use zeroes for final layer for stability (see FixUp Initialisation)
- Batch size – data is sparse so as high as possible (we used the full dataset)
- Optimiser – using AdamW
- Architecture: Neural networks are flexible and the structure can be varied to needs.
- Hyperparameter search – find best model parameters for Neurons in hidden layer, lasso penalty, weight decay, dropout
- “Rolling origin” cross validation
- Claim history feature engineering

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Comparison

On Simulated Data

Loss data

- Few examples of publicly available, detailed, real world data.
- Code for this presentation is fully available, so using simulated data.
- Five datasets from using a simulated package, SPLICE.
- Includes payments and reserves, but not exposures.
- Different behaviour for large vs attritional claims.

- "**Scenario 1**: simple, homogeneous claims experience, with zero inflation.
- **Scenario 2**: slightly more complex than 1, with dependence of notification delay and settlement delay on claim size, and 2% p.a. base inflation.
- **Scenario 3**: steady increase in claim processing speed over occurrence periods (i.e. steady decline in settlement delays).
- **Scenario 4**: inflation shock at time 30 (from 0% to 10% p.a.).
- **Scenario 5**: default distributional models, with complex dependence structures (e.g. dependence of settlement delay on claim occurrence period)."

From https://github.com/agi-lab/SPLICE/tree/main/datasets

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Results:

Dataset 5 Leaderboard

Method	Outstanding Claims Liability	Period Level MSE	Period Level Absolute Error	Total OCL Absolute Error	Total OCL Absolute Percent Error	
1	True Ultimate	415,208,224.0	0.0	0.0	0.0	0.0
7	D/O SplineNet	377,212,928.0	17,353,778.0	72,897,200.0	37,995,252.0	9.2
8	D/O SplineMDN	377,212,928.0	17,353,778.0	72,897,200.0	37,995,252.0	9.2
6	D/O ResNet	456,777,888.0	16,902,854.0	75,666,032.0	41,569,676.0	10.0
10	Detailed ResNet	478,930,661.1	41,708,402.1	157,432,251.7	63,722,452.3	15.3
11	Detailed SplineNet	483,915,060.2	42,700,794.0	161,107,670.6	68,706,851.4	16.5
12	Detailed SplineMDN	483,915,060.2	42,700,794.0	161,107,670.6	68,706,851.4	16.5
9	D/O SplineNet CV	548,784,064.0	31,238,146.0	146,933,376.0	133,575,840.0	32.2
2	Chain Ladder	553,335,232.0	43,015,804.0	174,280,848.0	138,127,024.0	33.3
3	GLM Chain Ladder	553,337,792.0	43,016,100.0	174,282,560.0	138,129,600.0	33.3
13	Detailed GBM	563,046,264.0	54,976,179.2	193,207,856.1	147,838,055.2	35.6
5	GLM Spline	565,708,352.0	35,842,056.0	164,710,784.0	150,500,096.0	36.2
4	GLM Individual	574,341,184.0	42,662,360.0	174,327,264.0	159,132,944.0	38.3
0	Paid to Date	0.0	102,023,488.0	415,208,224.0	415,208,224.0	100.0

NN’s do well for dataset 5: detailed features not leading to stronger predictions.

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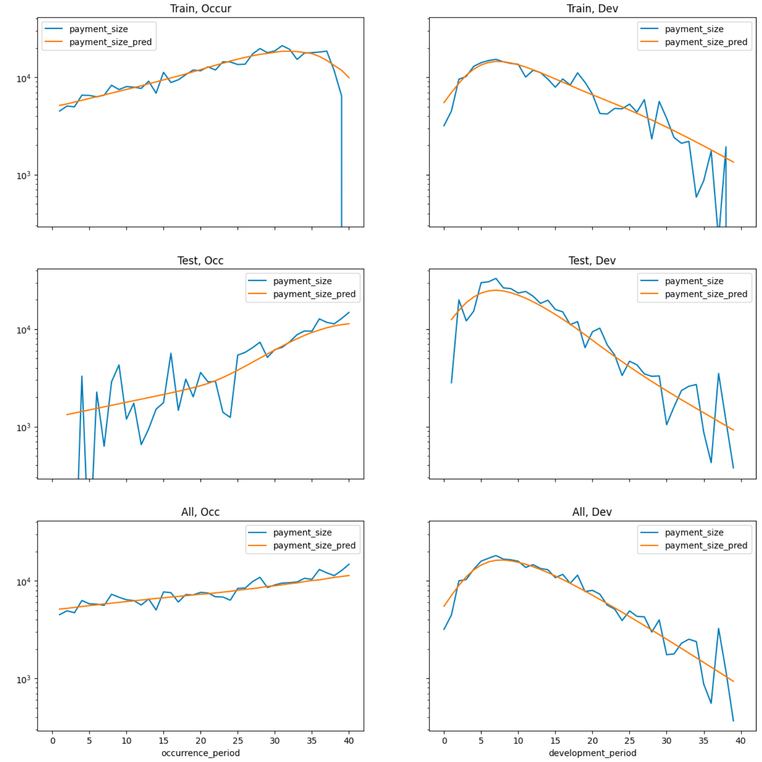
Customising the structure



- “SplineNet”, our customized design
- Using only occurrence and development periods only (but on individual data)
- Fits the development and occurrence trends across both train and test data

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Summary:

- Individual, granular models can be valuable in some circumstances
- Neural networks can effectively model trends in claims data:
  - Reflect trends
  - Potential to use detailed claims information
  - Probabilistic output
- Link for full details: https://institute-and-faculty-of-actuaries.github.io/mlr-blog/post/research/chain_ladder_to_individual_mdn/

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