Individual reserving – a survey

R in Insurance, Paris 08-June-2017
Alexandre Boumezoued, joint work with Laurent Devineau
## Agenda

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The current reserving practice consists, in most cases, of using methods based on **claims development triangles** for point estimate projections as well as for capital requirement calculations.

- The triangles are organised by origin (e.g., accident, underwriting) and development period.

In the context of an increasing need within the reserving practice for more accurate models, taking advantage of the **information embedded in individual claims data** is a promising alternative compared with the traditional aggregate triangles.

Traditional reserving methods have worked well in several circumstances in the past.

- Today, however, the awareness of the insurance market about some possible **limitations of traditional aggregate models** to provide robust and realistic estimates in more variable contexts has reached a level which should be noted.

Several potential limits of aggregate models based on triangles have indeed already been highlighted both from a practical and a theoretical point of view:

- **Over/under-estimation** of the distribution when back-testing realised amounts with forecasts
- **Huge estimation error** for the latest development periods due to the lack of observed aggregate amounts
- Uncertainty about the ability of these models to properly capture the **pattern of claim development**, combined with the **limited interpretive and predictive power** of the accident and development period parameters.
As noted in the report on worldwide non-life reserving practices from the ASTIN Working Party on Non-Life Reserving (June 2016), there is ‘an increase in the need to move towards individual claims reserving and big data, to better link the reserving process with the pricing process and to be able to better value non-proportional reinsurance.’

It is interesting to note that stochastic models for unpaid claims reserving appeared at around the same time for both individual-based and triangle-based models.

- To our knowledge, Norberg (1983, 1993, 1999), Jewell (1987), Arjas (1989) and Hesselager (1994) are among the earliest papers which introduced a proper probabilistic setting for individual claims reserving, recently applied by Antonio and Plat (2014)

- To be compared with the stochastic models for triangles in Mack (1993) and following contributions

To date, we suspect that the greater success of the triangle-based models could be driven by their comparative ease of use (true?) and the lack of inexpensive computing power in the early days of these models.
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Model
General parametrization of the individual model

- The individual claims paths are modelled with continuous time stochastic processes

Occurrence Reporting Payments Settlement

$t \leftarrow u \rightarrow v$ Time

Claims occur at times $T_n$ according to some Poisson process with intensity $\lambda(t)$

Claims are reported with a delay with distribution $p_{U|t}(du)$

Payments and settlement events are modelled using three types of events:

- (1) settlement without payment at settlement
- (2) settlement with payment at settlement
- (3) payment without settlement

Each type of event (1,2,or 3) occurs according to its specific intensity parameter $h_1(v)$, $h_2(v)$ or $h_3(v)$:

can be seen as a recursive competing risks model

If an event $i \in \{2,3\}$ occurs $v$ time units after reporting, then random payments $Y_i(v)$ are generated

Hesselager (1994)
Antonio & Plat (2014)
Model

Poisson point measure representation

- **Poisson point measure (PPM):** A powerful tool to study Marked Poisson processes
  - Defined on $\mathbb{R}_+^2$ as $Q(dt, du) = \sum_{n \geq 1} \delta_{(t_n, u_n)}(dt, du)$, it has intensity measure $\lambda(t)dt p_{U|t}(du)$
  - Example: $\int_0^\tau \int_0^\infty Q(dt, du) = \sum_{n \geq 1} 1_{t_n \leq \tau}$ is the number of claims which occurred before time $\tau$
- **Key property 1:** for measurable $A \subset \mathbb{R}_+^2$, $Q(A)$ is a Poisson random variable with parameter $\int_A \lambda(t)dt p_{U|t}(du)$
- **Key property 2:** if $A \cap B = \emptyset$, then the random variables $Q(A)$ and $Q(B)$ are independent
- **Key property 3:** The events frequency in the set $A$ can be recovered as $\int_{u:(t,u)\in A} \lambda(t) p_{U|t}(du)$

- **Example:** the number of IBNyR at time $\tau$ writes
  $$ N^{IBNyR}_\tau = \int_0^\tau \int_0^\infty Q(ds, du) $$

- Let us denote by $X^{(s)}(t_1, t_2)$ the total payments for claim occurred at time $s$ between $t_1$ and $t_2$ time units after occurrence
- **Example:** the IBNyR future payments write
  $$ X^{IBNyR}_\tau = \int_0^\tau \int_{\tau-t}^\infty X^{(s)}(0, \infty) Q(ds, du) $$
Agenda

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Results
Micro-macro consistency

- What is the **aggregate dynamics** resulting from the micro model?
- Let us introduce the incremental number triangle as
  \[ X_{i,j} = \sum_{n \geq 1} 1_{T_n \in [i,i+1]} 1_{U_n \in [j,j+1]} = \int_i^{i+1} \int_{j-t}^{j+1-t} Q(dt, du) \]
- **Key property 1** shows that \( X_{i,j} \) is Poisson distributed with parameter
  \[ \int_i^{i+1} \int_{j-t}^{j+1-t} \lambda(t)dt \, p_{U|t}(du) \]
- **Key property 2** shows that the \((X_{i,j})\) are independent
- **Key property 3** shows that the occurrence intensity of **reported claims** is
  \[ t \mapsto \lambda(t) p_{U|t}([0, \tau - t]) \]

This shows that the related triangle is governed by the Poisson model.

Useful to derive the likelihood.
Results
Simulation of the claims population

- The thinning procedure is a powerful simulation tool to draw future claims paths with general time-dependent frequency parameters
  - Example below: simulation of a non-homogeneous Poisson process

- The intensity $\lambda(t)$ being given, one has to simulate a Poisson process with such intensity
  - **Thinning procedure**: assume that this intensity is bounded, that is $\lambda(t) \leq \bar{\lambda}$

-> One is able to easily simulate a Poisson process with intensity $\bar{\lambda}$ as a sequence of $(\bar{T}_n)_{n \geq 1}$ such that the $(\bar{T}_n - \bar{T}_{n-1})$ are iid exponentially distributed with parameter $\bar{\lambda}$

-> Then, select each occurrence $\bar{T}_n$ with probability $\lambda(\bar{T}_n) / \bar{\lambda}$
Practical illustration
Data set

Occurrence times

Reporting delays

Time

Delay (months)

Frequency

Frequency
# Practical illustration
Forecasting the IBNyR: micro vs macro

<table>
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<tr>
<th>Model</th>
<th>Expected IBNyR</th>
<th>Process error</th>
<th>Estimation error</th>
<th>Prediction error</th>
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<tr>
<td>Mack Chain-Ladder model</td>
<td>328</td>
<td>32.5</td>
<td>107.4</td>
<td>112.3</td>
</tr>
<tr>
<td>Individual claims model</td>
<td>217</td>
<td>14.7</td>
<td>2.2</td>
<td>14.9</td>
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- **Reduction in process error:** the individual model takes advantage of its Poisson macro-consistency
- **Reduction in estimation error:** the individual model takes advantage of the large amount of individual data
- **Overall reduction in prediction error:** the use of the individual model reduces reserves uncertainty
Practical illustration
Forecasting the IBNyR: micro vs macro

Use of Mack Chain Ladder of the ‘ChainLadder’ Package

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THEY DIDN’T TEACH THIS IN BUSINESS SCHOOL.
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