Centre for
Data Analytics

Insight

General Insurance Claims Modelling with Factor Collapsing and Bayesian Model Averaging

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Motivation:

- Model uncertainty with variable selection $\Rightarrow$ how confident we should be about the final model
- Existence of high multi-level factors - a factor having too many levels for a GLM structure $\Rightarrow$ model parsimony and interpretability issues
  - lack of sufficient number of observations
  - insignificant levels should be merged (too many parameters)

2 questions to answer:

- Which categorical predictors should be included in the model?
- Which categories within one categorical predictor should be distinguished?

Motivation

Factor collapsing (FC) assesses the optimal manner of categories: which differs from one another w.r.t dependent variable $\Rightarrow$ uncertainty about the optimal manner

Bayesian model averaging (BMA) takes such model uncertainty into consideration:

- variable selection uncertainty
- factor level selection uncertainty

Example: a question from "faraway" package [1]

Standard GLM output in R, for "Make" predictor in frequency model

	Estimate	Std. Error	z value	Pr(>|z|)	
(Intercept)	-1.812178	0.013758	-131.721	< 2e-16	***
...					
Make2	0.086384	0.021240	4.067	4.76e-05	***
Make3	-0.226013	0.025098	-9.005	< 2e-16	***
Make4	-0.640736	0.024196	-26.481	< 2e-16	***
Make5	0.161510	0.020235	7.982	1.44e-15	***
Make6	-0.331235	0.017375	-19.063	< 2e-16	***
Make7	-0.044705	0.023344	-1.915	0.0555	.
Make8	-0.008300	0.031606	-0.263	0.7929	
Make9	-0.069596	0.009956	-6.990	2.74e-12	***

Standard GLM output in R, for "Kilometres" predictor in severity model

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	8.39456	0.02335	359.474	< 2e-16	***
...					
Kilometres2	0.02455	0.01290	1.903	0.05718	.
Kilometres3	0.02124	0.01487	1.429	0.15327	
Kilometres4	0.04306	0.02073	2.077	0.03793	*
Kilometres5	0.03945	0.02205	1.789	0.07374	.

Factor collapsing

Set partition: grouping elements within a set into non-empty subsets, in such a way that every element is included in one and only one subsets. ("partitions" R package [2])

Partitioning 3-element set $\{1, 2, 3\}$:

$$\{\{1\}, \{2\}, \{3\}\}$$
$$\{\{1, 2\}, \{3\}\}$$
$$\{\{1, 3\}, \{2\}\}$$
$$\{\{1\}, \{2, 3\}\}$$
$$\{\{1, 2, 3\}\} \Rightarrow \text{variable removed}$$

Fit each (combination of) partition into a pre-specified model
Bell number increases nearly exponentially

BMA

Use BMA to average the best models (where possible)

$$Pr(\Delta|D) = \sum_{k=1}^{K} Pr(\Delta|M_k, D)Pr(M_k|D) \tag{1}$$

$$P(M_k|D) \approx \frac{\exp(-.5BIC_k)}{\sum_{r=0}^{K} \exp(-.5BIC_r)} \tag{2}$$

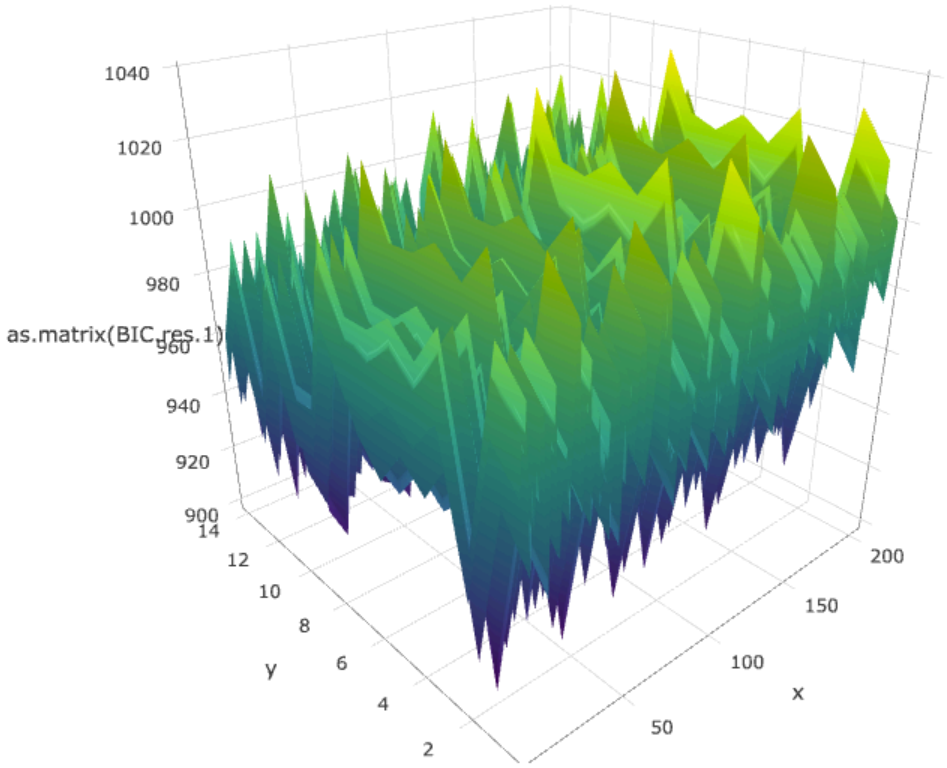
- Average over model prediction
- Average over model coefficients

Stochastic search

Number of set partitions increases nearly exponentially

$\Rightarrow$ computationally intensive

$\Rightarrow$ **it becomes an optimisation problem**



Simulated Annealing

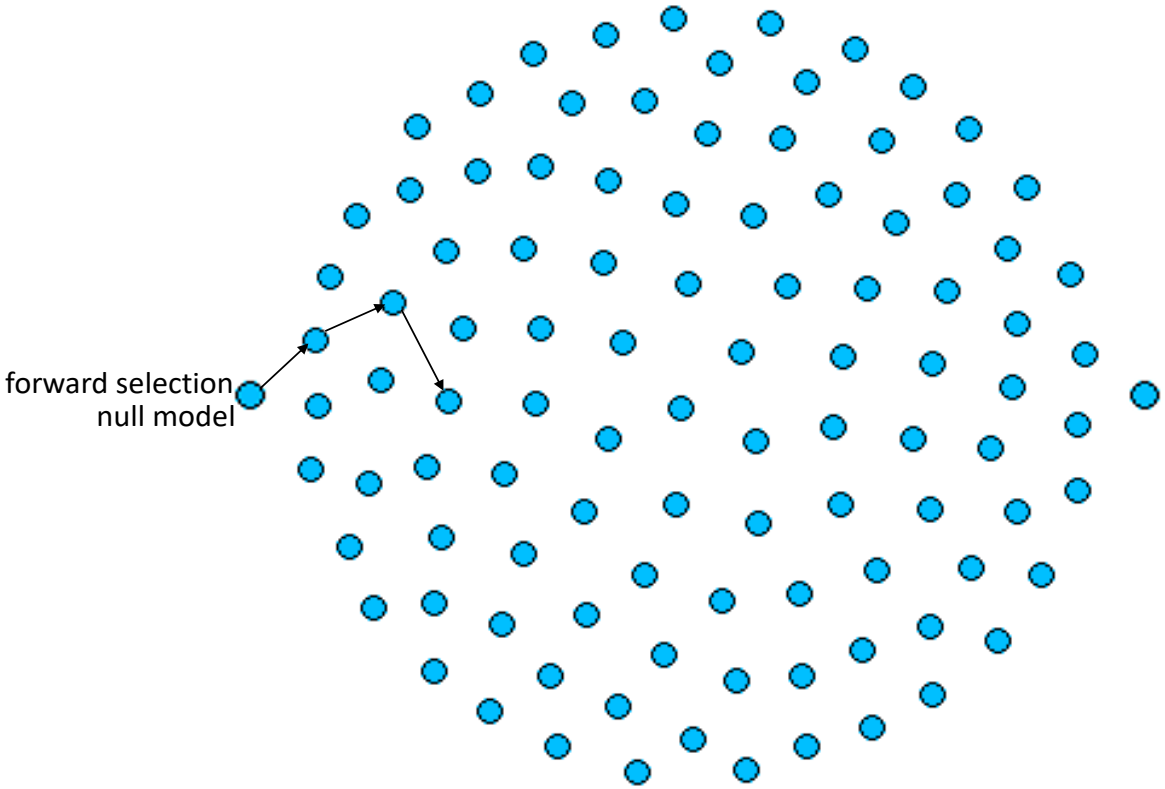
Global optimisation technique based on Monte Carlo method, similar to the MC^3 technique proposed in Hoeting et al. (1999) [3].

- Starting from a random state
- Make random state changes, accepting worse moves with probability determined by temperature
- Reduce temperature after reaching (close-to) equilibrium
- Stop once temperature gets very small

Other stochastic optimisation methods also work for this non-linear non-differentiable objective function, such as genetic algorithm etc.

FC-BMA illustration

Comparing FC-BMA with stepwise selection using BIC/AIC:



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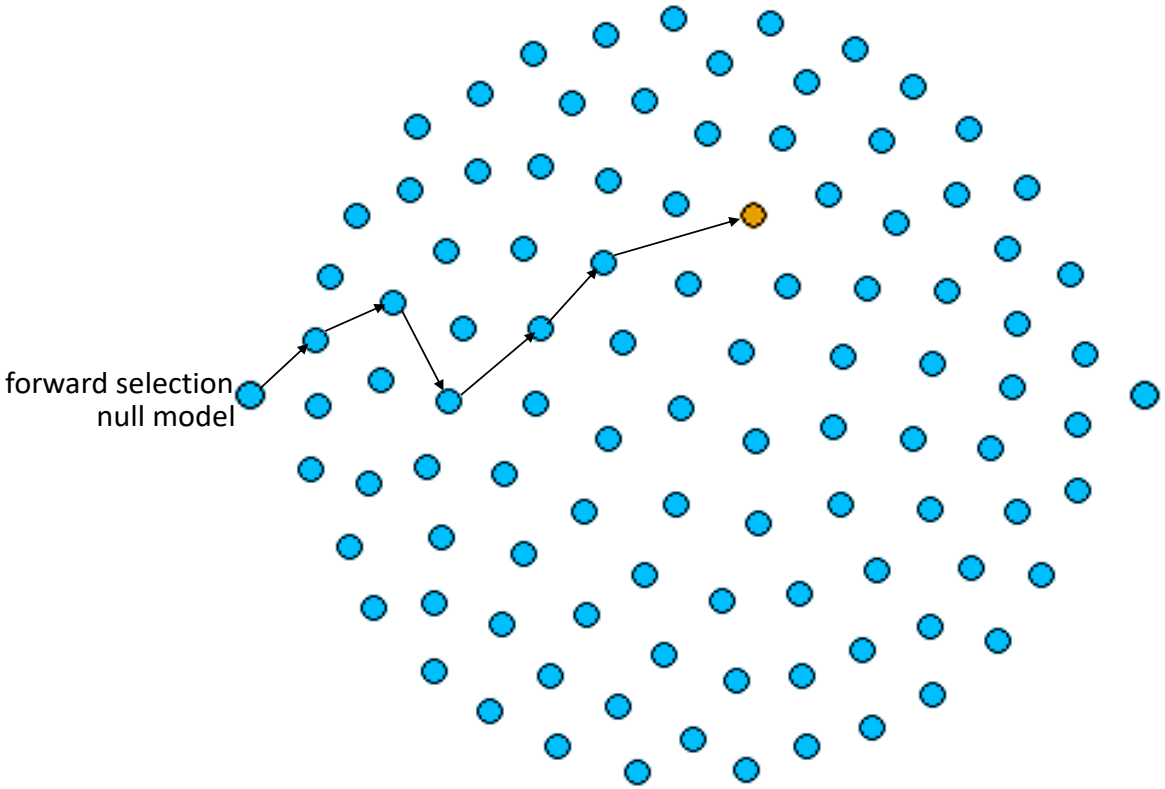

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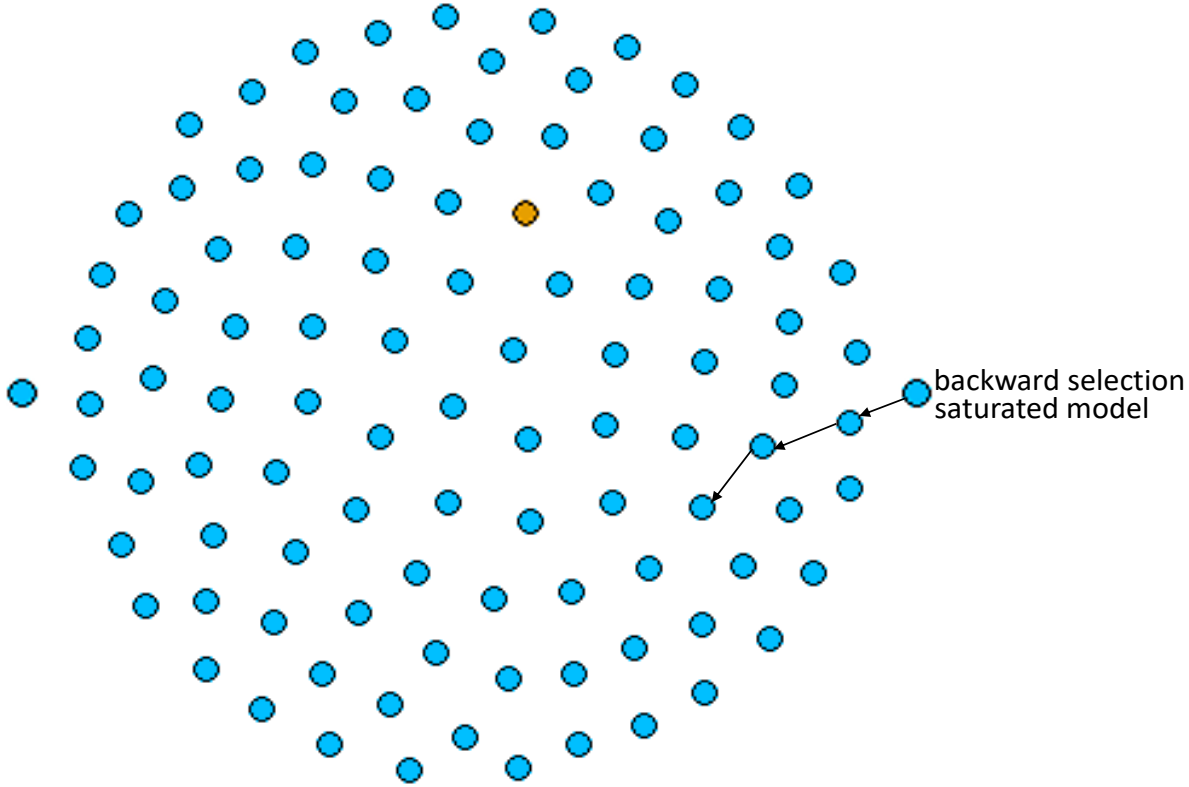

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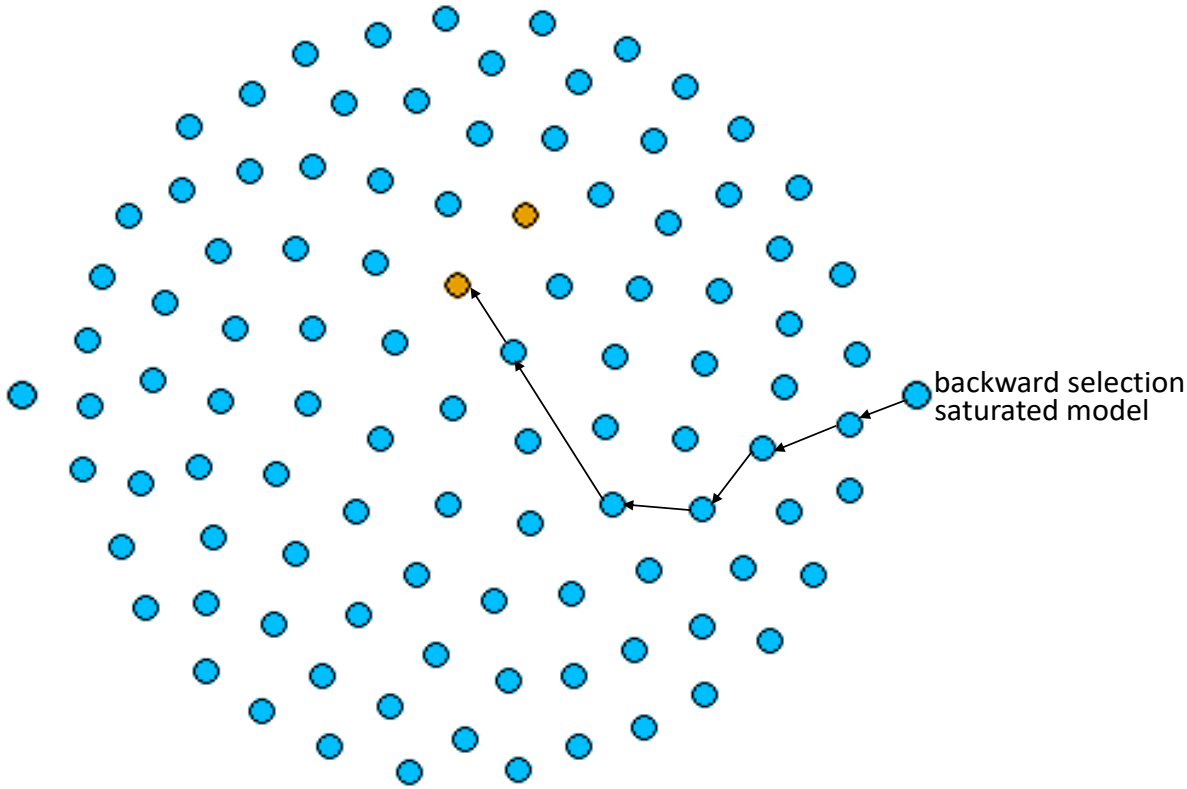

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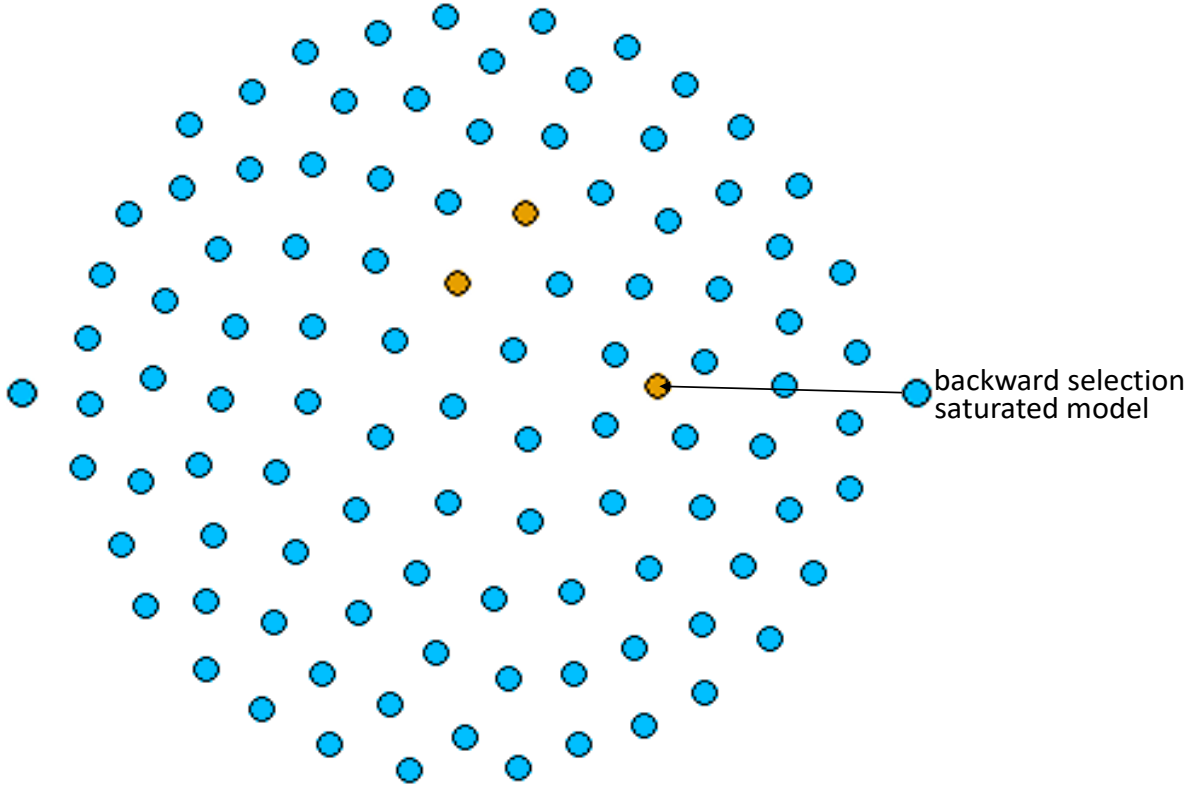

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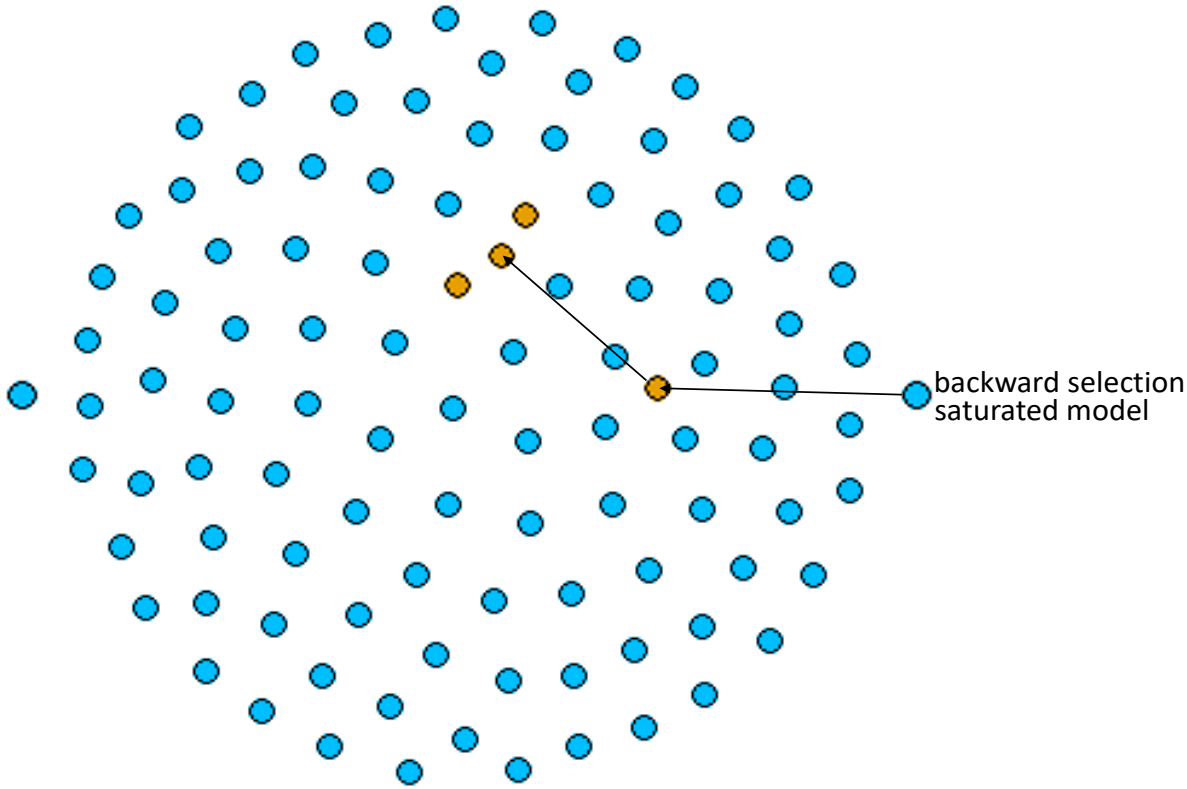

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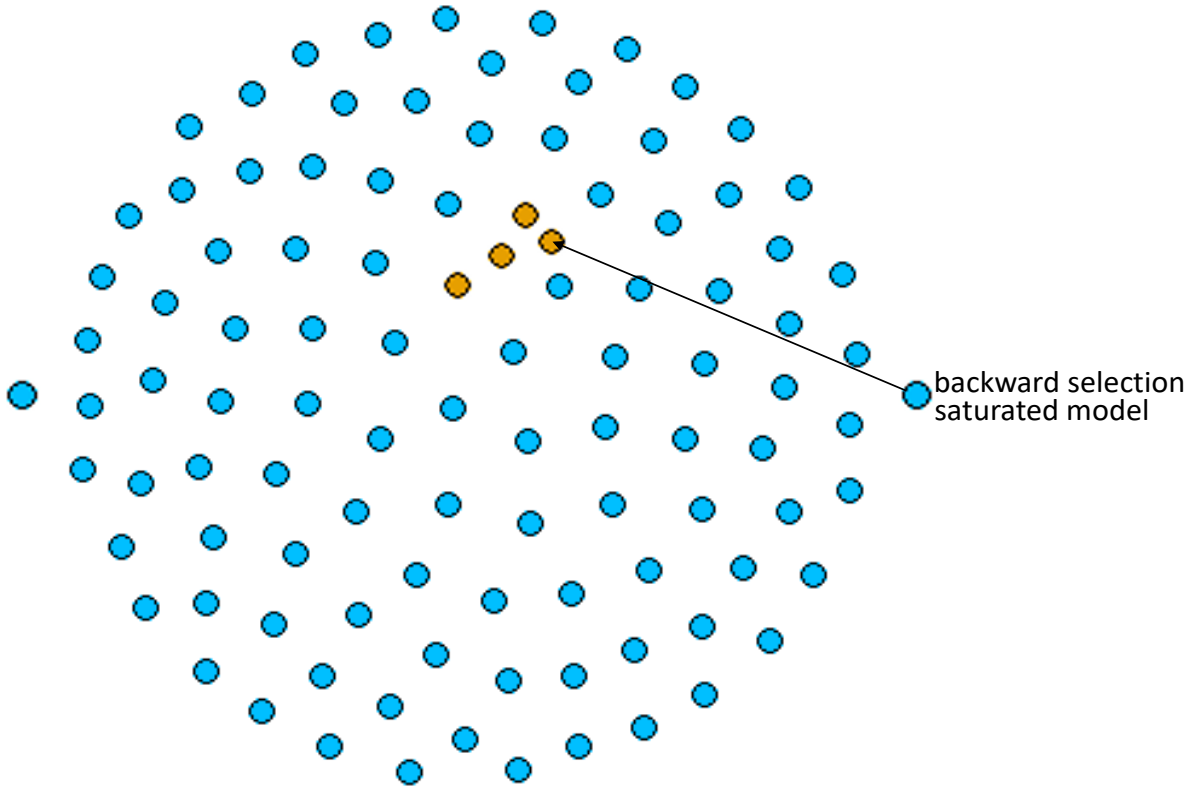

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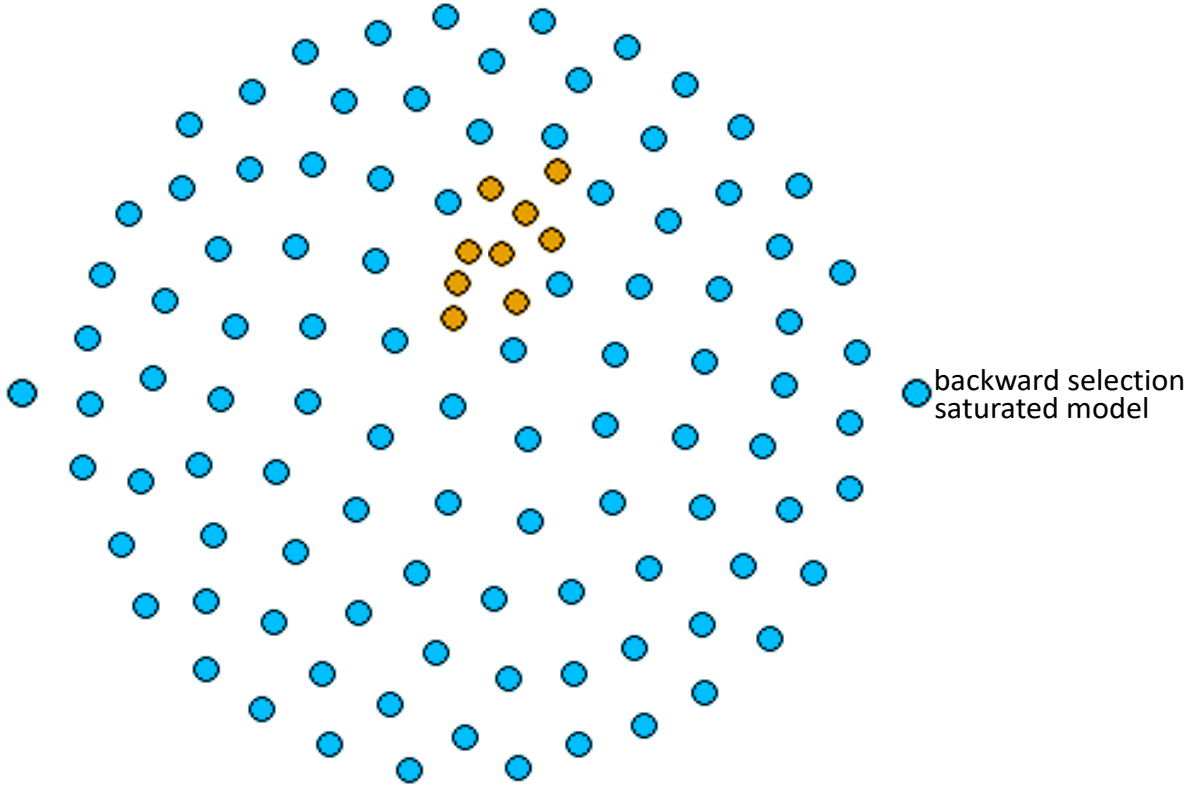

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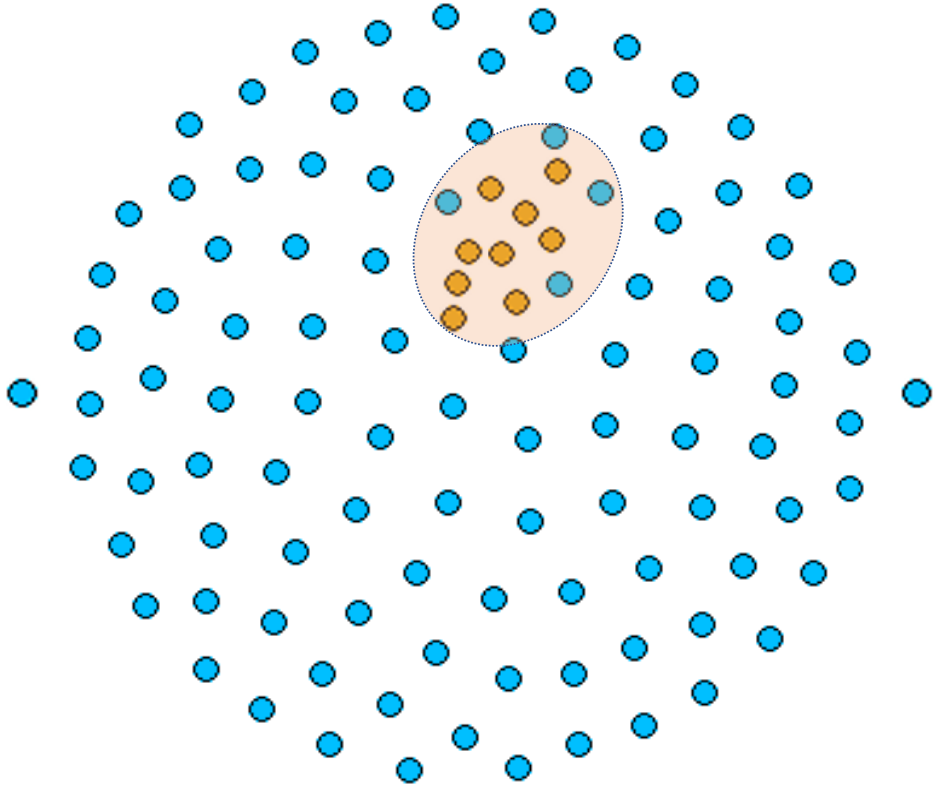

Following up the example...

Table: Results for collapsing "Make" factor only in frequency model. Here only the best 5 models (based on their BIC values) are shown.

Make: 1, 2, 3, 4, 5, 6, 7, 8, 9		
combination	BIC	BMA weight
(1,8)(2)(3)(4)(5)(6)(7,9)	10301.11	0.34579
(1,8)(2,5)(3)(4)(6)(7,9)	10301.81	0.24257
(1,7,8)(2)(3)(4)(5)(6)(9)	10303.44	0.10764
(1,7,8)(2,5)(3)(4)(6)(9)	10304.15	0.07541
(1)(2)(3)(4)(5)(6)(7,8,9)	10304.92	0.05136

Following up the example...

Table: Result for collapsing "Kilometres" factor only in severity model, only the best 5 models (based on BIC values) are shown.

Kilometres: 1, 2, 3, 4, 5		
combinations	BIC	BMA weight
(1)(23)(45)	1874293	0.90779
(1)(2)(3)(45)	1874299	0.05977
(1)(23)(4)(5)	1874300	0.03043
(1)(2)(3)(4)(5)	1874305	0.00200
(1)(25)(3)(4)	1874338	0.00000

Irish counties

Irish county level clustering with an Irish GI insurer:

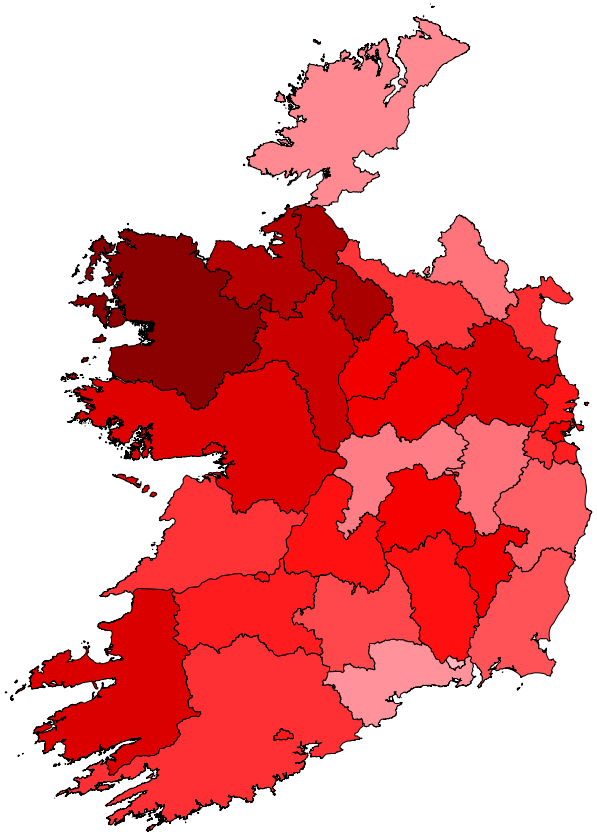

Figure: Frequency

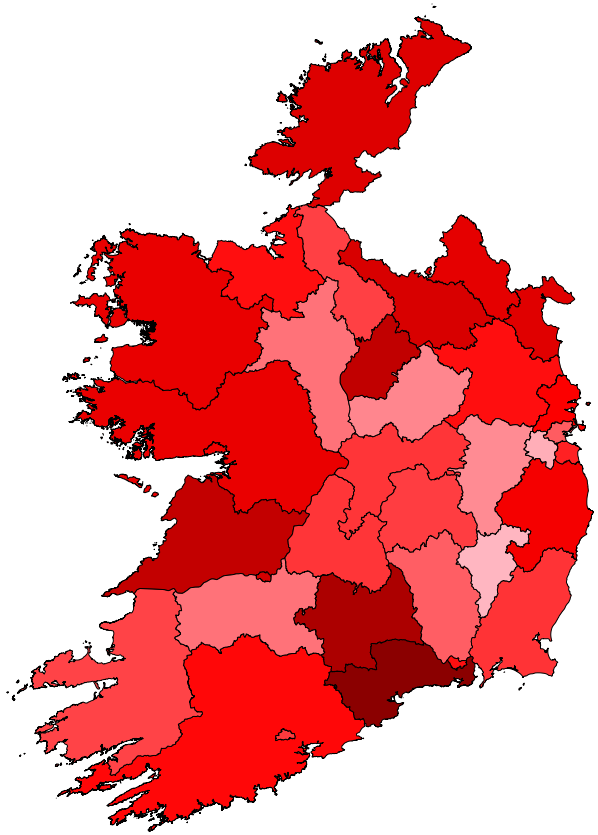

Figure: Severity

County	model coef.	new coef.
Waterford City	-6.6556	-6.6415
Unknown	-6.6130	-6.6415
Waterford County	-6.6073	-6.6415
Donegal County	-6.5959	-6.6415
Offaly County	-6.5787	-6.5733
Monaghan County	-6.5670	-6.5733
Kildare County	-6.5638	-6.5733
Wicklow County	-6.5397	-6.5733
Wexford County	-6.5217	-6.5733
South Tipperary	-6.5063	-6.5001
Cavan County	-6.4809	-6.5001
Clare County	-6.4764	-6.5001
Cork County	-6.4738	-6.5001
Louth County	-6.4720	-6.5001
South Dublin	-6.4708	-6.5001
Dun Laoghaire-Rathdown	-6.4489	-6.4648
Limerick County	-6.4473	-6.4648
Cork City	-6.4385	-6.4648
Fingal	-6.4379	-6.4648
North Tipperary	-6.4323	-6.4648
Limerick City	-6.4306	-6.4648
Kilkenny County	-6.4299	-6.4648
Laois County	-6.3923	-6.3766
Carlow County	-6.3865	-6.3766
Longford County	-6.3813	-6.3766
Westmeath County	-6.3808	-6.3766
Dublin City	-6.3694	-6.3766
Galway City	-6.3421	-6.3766
Galway County	-6.3415	-6.3766
Kerry County	-6.3323	-6.3766
Meath County	-6.3282	-6.3766
Roscommon County	-6.3031	-6.3766
Sligo County	-6.2503	-6.2106
Leitrim County	-6.2282	-6.2106
Mayo County	-6.1615	-6.2106

Irish counties

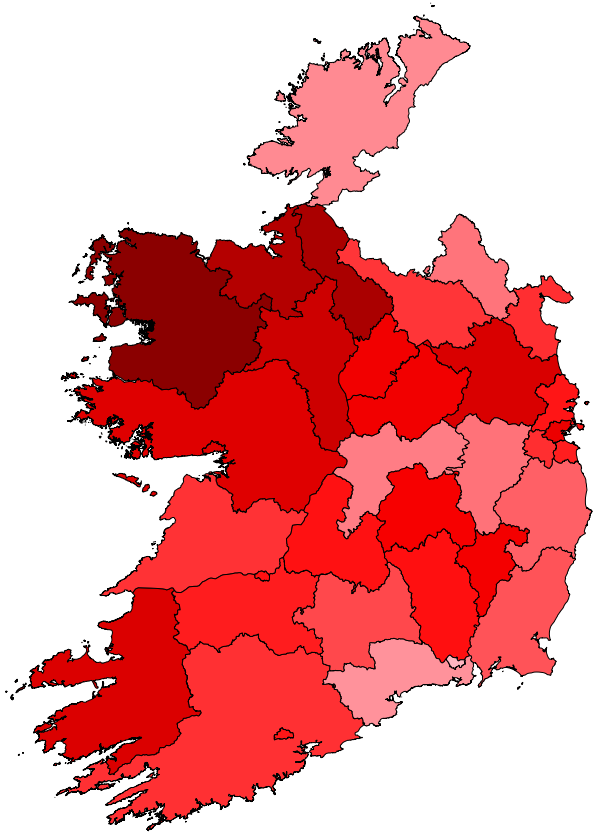

Figure: Frequency: before clustering

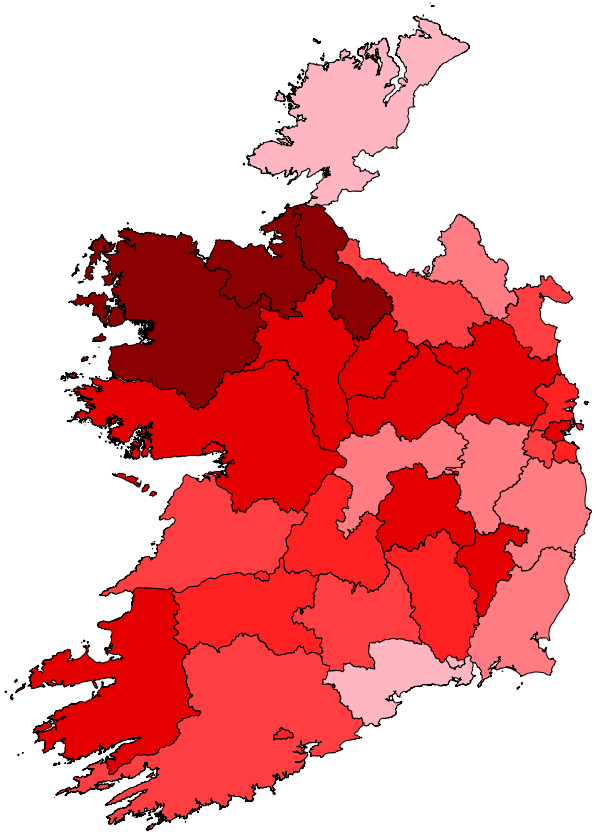

Figure: Frequency: after clustering

Table: (Subset of) Frequency model coefficients for the baseline standard GLM, and results of FC-BMA. Categorical levels are of increasing order based on the standard GLM. Only 5 are selected here for illustration.

	Std. GLM	BMA	Model 1	Model 2	Model 3	Model 4	Model 5
BIC			62807.2927	62807.3039	62807.3972	62807.4069	62807.4294
Model weights of all selected models			0.0233	0.0232	0.0221	0.0220	0.0218
Model weights of the 5 models			0.2074	0.2062	0.1968	0.1959	0.1937
Waterford City	-6.6556	-6.6359	-6.6414	-6.6399	-6.6326	-6.6341	-6.6311
Unknown	-6.6130	-6.6359	-6.6414	-6.6399	-6.6326	-6.6341	-6.6311
Waterford County	-6.6073	-6.6359	-6.6414	-6.6399	-6.6326	-6.6341	-6.6311
Donegal County	-6.5959	-6.6359	-6.6414	-6.6399	-6.6326	-6.6341	-6.6311
Offaly County	-6.5787	-6.6218	-6.5733	-6.6399	-6.6326	-6.6341	-6.6311
Monaghan County	-6.5670	-6.6080	-6.5733	-6.5732	-6.6326	-6.6341	-6.6311
Kildare County	-6.5638	-6.5695	-6.5733	-6.5732	-6.5689	-6.5674	-6.5645
Wicklow County	-6.5397	-6.5695	-6.5733	-6.5732	-6.5689	-6.5674	-6.5645
Wexford County	-6.5217	-6.5695	-6.5733	-6.5732	-6.5689	-6.5674	-6.5645
South Tipperary	-6.5062	-6.5263	-6.5000	-6.5023	-6.5006	-6.5674	-6.5645
Cavan County	-6.4809	-6.5004	-6.5000	-6.5023	-6.5006	-6.5011	-6.4980
Clare County	-6.4764	-6.5004	-6.5000	-6.5023	-6.5006	-6.5011	-6.4980
Cork County	-6.4738	-6.5004	-6.5000	-6.5023	-6.5006	-6.5011	-6.4980
Louth County	-6.4720	-6.5004	-6.5000	-6.5023	-6.5006	-6.5011	-6.4980
South Dublin	-6.4708	-6.5004	-6.5000	-6.5023	-6.5006	-6.5011	-6.4980

Table: Prediction comparison in Swedish TPML dataset, using MSE, Gini index, concordance correlation coefficient (CCC), Wasserstein distance, Kolmogorov-Smirnov test (KS-test), KL divergence respectively.

80% and 20% split		MSE	Gini	CCC	Wass.	KS-test	KL
Frequency	no FC-BMA	266.9408	0.8266	0.9968	3.0340	0.0736(0.3045)	0.0122
	FC-only	224.7803	0.8267	0.9943	2.9696	0.0788(0.2358)	0.0114
	FC-BMA(5)	456.3766	0.8267	0.9973	4.2012	0.0778(0.2535)	0.0113
Severity	no FC-BMA	14748455	0.0567	0.0409	1948.3340	0.4489(0)	0.2191
	FC-only	14664567	0.0576	0.0667	1825.0540	0.4067(0)	0.2178
	FC-BMA(5)	14666355	0.0576	0.0657	1822.9450	0.4033(0)	0.2178

Summary

- FC-BMA deals with model selection and uncertainty, categorical level selection simultaneously.
- It helps improve the model parsimony, interpretability, and prediction.
- Compared with other existing methods in literature, it does not require deciding extra parameters.
- It can be a challenge to obtain the optimum through stochastic optimisation, and may take a long time to reach the optimum.

References

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Q & A...