LTC insurance with markovchain

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Intro

LTC insurance

- As well known, Long Term Care (LTC) policies guarantee annuity benefits as long as a disability status is present.
- The actuarial approach to model LTC insurances have been traditionally based on the multi - state approach.
- The demographic assumptions are represented by tables that model the transition between (A)ctive (healthy), disabled/(I)ll and (D)ead status across ages.

- The lump-sum premium for a yearly benefit C is $U = \sum_{h=1}^{\omega-x} p_{x,h-1}^{hh} * q_{x+h-1}^{hi} * v^h * \ddot{a}_{x+h}^{(i)} = P * \ddot{a}_x^{(h)}$.
- The annual premium is conventionally paid when the policyholder is (H)ealthy.
- Reserves depends by the attained status (H or I) at the evaluation period.

Empirical data

- We used assumptions for the Italian population taken from (Paolo de Angelis 2016).
- In particular, this exercise is based on the transition probabilities for Italian male population estimated for the 2016 calendar year.
- Possible states are (A)ctive, (I)ll and (D)ead. Modeled transitions are from A -> I, from A -> D and I -> D.

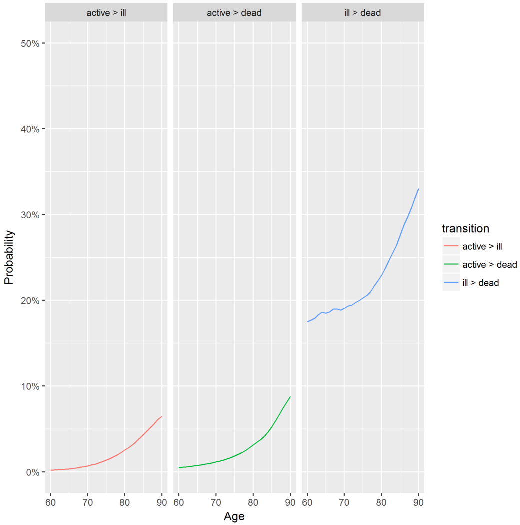


Figure 1: Italian Males Transition Probabilities 2016

The markovchain package

Purpose

- Package created for easily handling Discrete Time Markov Chains (DTMC) in R by S4 classes for homogeneous and not DTMCs.
- Also used to perform structural analysis (e.g. states classification), statistical inference, estimation and simulation.
- The functions written to perform the actuarial analysis on LTC data heavily relies on the markovchain package's simulation functions.

Package history

- On Cran since mid 2013.
- Development sponsored by Google within the Google Summer of Code 2015, 2016 and 2017.
- Core parts written in Rcpp (Eddelbuettel 2013). The simulation function also uses RcppParallel (Allaire et al. 2016).

Application to LTC insurance

- The stochastic process underlying a LTC insurance can be considered a non - homogeneous DTMC, since transition probabilities vary by age.

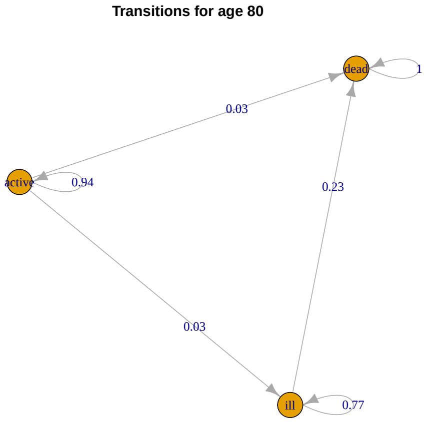
```
#defining the transition diagram for age 80
mc80<-createAgeMarkovChain(80)
mc80
```

```
80
 A  3 - dimensional discrete Markov Chain defined by the following states:
 active, ill, dead
 The transition matrix  (by rows)  is defined as follows:
          active        ill       dead
active 0.9432764 0.02541479 0.03130879
ill    0.0000000 0.77097391 0.22902609
dead   0.0000000 0.00000000 1.00000000
```

```r
#summarizing the structural proprieties of
#the transition diagram
summary(mc80)
```

```
80  Markov chain that is composed by:
Closed classes:
dead
Recurrent classes:
{dead}
Transient classes:
{active},{ill}
The Markov chain is not irreducible
The absorbing states are: dead
```

```r
#plotting
plot(mc80,main="Transitions for age 80")
```



- The markovchain package allows to draw samples from non - homogenous DTMC.

```
# simulating life trajectories
table90 <- getTable(age = 90)
simulateLifeTrajectiories(transitionTable = table90, numSim =
```

```
  91        92        93     94     95     96     97     98     99
1 "active" "active" "ill" "ill" "ill" "dead" "dead" "dead" "de
  101    102    103    104    105    106    107    108    109
1 "dead" "dead" "dead" "dead" "dead" "dead" "dead" "dead" "dea
  111    112    113    114    115    116    117    118    119
1 "dead" "dead" "dead" "dead" "dead" "dead" "dead" "dead" "dea
  121    122
1 "dead" "dead"
```

Actuarial analysis of LTC coverages with R

Assumptions

- A simulation approach is used to actuarially evaluate the LTC coverage.
- The (fictionary) policyholder age is 75, the real interest rate is $i = 0.01$.
- we will compute benefit premiums (lump sum and yearly) and reserves.

- A large number of random life trajectories is sampled.
- Cash flows depends by the status at the beginning of the year, 1 when (D)isabled, 0 otherwise.
- The yearly benefit is set to 12K.

Simulating life trajectories

```
## retrieving transition since age 75
table.75<-getTable(75)
## simulating life trajectories
lifetrajectories.75<-simulateLifeTrajectiories(transitionTable
  numSim = 1000,
  include_start_age = FALSE,
  begin_status = "active")
```

```r
#sampled life trajectories for a policyholder aged 75
lifetrajectories.75[1:5,10:15]
```

```
  85       86       87       88       89       90
1 "dead"   "dead"   "dead"   "dead"   "dead"   "dead"
2 "active" "dead"   "dead"   "dead"   "dead"   "dead"
3 "dead"   "dead"   "dead"   "dead"   "dead"   "dead"
4 "active" "active" "active" "active" "active" "active"
5 "active" "active" "active" "active" "active" "active"
```

```r
#computing expected future years disabled
disabled01<-matrix(0,nrow=nrow(lifetrajectories.75),
ncol=ncol(lifetrajectories.75))
disabled01[which(lifetrajectories.75=="ill")]=1
mean(rowSums(disabled01))
```

```
[1] 1.313
```

Computing premiums

```
#PV of a policy paying 12K euro at the beginning of the year
#if the policyholder is disabled
##simulating
pvbenefits.75<-getPVDistribution(lifetrajectoriesMatrix
= lifetrajectories.75, target = "ill",CF = 1000*12,
real_interest_rate = 0.01,
begin = 1)
```

```r
## computing APV of lump sum benefits
U<-mean(pvbenefits.75); U
```

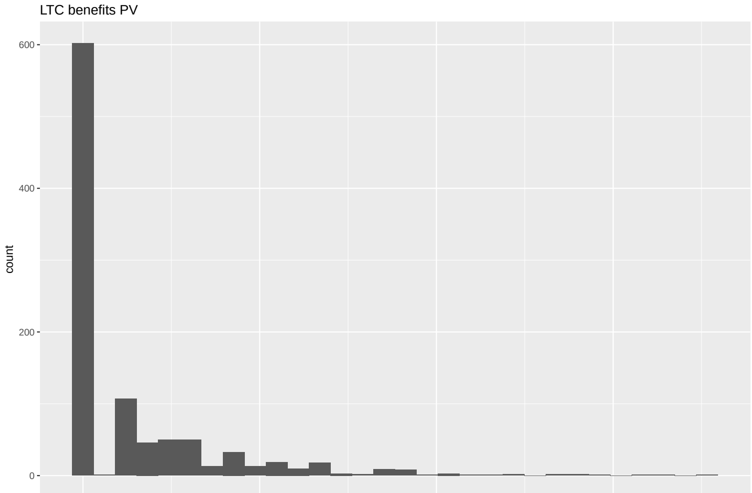
```
[1] 13983.72
```

```r
## yearly premium
annuity.75.healthy<-mean(getPVDistribution(lifetrajectoriesMatrix =
lifetrajectories.75, target = "active", CF = 1, real_interest_rate = 0.01,
P <- U/ annuity.75.healthy;P
```

```
[1] 1383.38
```

- It is also possible to perform a stochastic analysis of the future benefits distribution

```
qplot(pvbenefits.75,main = "LTC benefits PV",xlab="Euros")
```



Computing reserves

- A prospective approach is used.
- Reserves depend by attained status (Healthy or Ill)
- Calculations performed at example age of 80.

```
## retrieving transition from age 80
table.80<-getTable(80)
## simulating life trajectories (for healthy insureds)
lifetrajectories.80.healthy<-simulateLifeTrajectiories(transitionTable = ta
  numSim = 1000,
  include_start_age = FALSE,
  begin_status = "active")
## (for ill insureds)
lifetrajectories.80.ill<-simulateLifeTrajectiories(transitionTable = table.
  numSim = 1000,
  include_start_age = FALSE,
  begin_status = "ill")
```

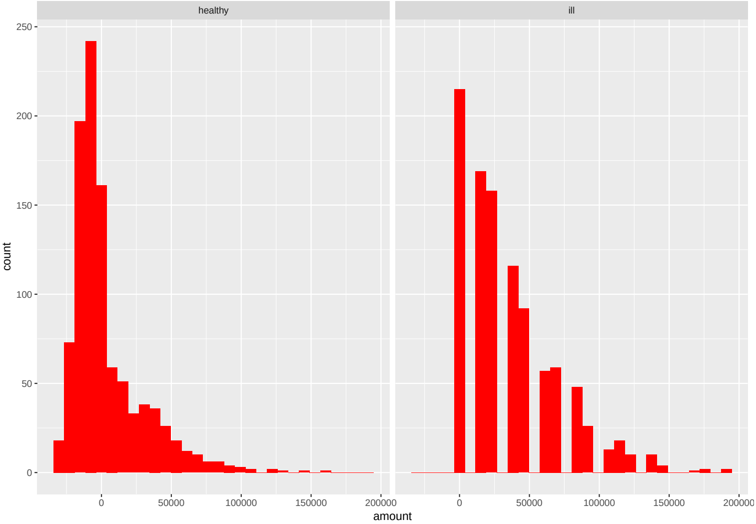
```
## benefit reserve distribution, 80 yo healthy
reserve.80.active.distr<-
  getReserveDistribution(lifetrajectoriesMatrix = lifetrajectories.80.healt
  CF_active = -P,CF_ill = +12000,CF_dead = 0,real_interest_rate = 0.01,begi
mean(reserve.80.active.distr)
```

```
[1] 3774.539
```

```
## benefit reserve distribution, 80 yo ill
reserve.80.ill.distr<-
  getReserveDistribution(lifetrajectoriesMatrix = lifetrajectories.80.ill,
  CF_active = -P,CF_ill = +12000,CF_dead = 0,real_interest_rate = 0.01,begi
mean(reserve.80.ill.distr)
```

```
[1] 35636.3
```

- It is also possible to quantify reserves' variability



References

Allaire, JJ, Romain Francois, Kevin Ushey, Gregory Vandenbrouck, Marcus Geelnard, and Intel. 2016. *RcppParallel: Parallel Programming Tools for 'Rcpp'*. `https://CRAN.R-project.org/package=RcppParallel`.

Eddelbuettel, Dirk. 2013. *Seamless R and C++ Integration with Rcpp*. New York: Springer.

Paolo de Angelis, Luigi di Falco. 2016. *Assicurazioni Sulla Salute: Caratteristiche, Modelli E Basi Tecniche*. Il mulino.