LTC insurance with markovchain

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LTC insurance

- As well known, Long Term Care (LTC) policies guarantee annuity benefits as long as a disability status is present.
- The actuarial approach to model LTC insurances have been traditionally based on the multi-state approach.
- The demographic assumptions are represented by tables that model the transition between (A)ctive (healthy), disabled/(I)ll and (D)ead status across ages.
The lump-sum premium for a yearly benefit $C$ is
$$U = \sum_{h=1}^{\omega-x} p_{x,h-1}^{hh} * q_{x+h-1}^{hi} * v^h * \ddot{a}_{x+h}^{(i)} = P * \ddot{a}_x^{(h)}.$$

The annual premium is conventionally paid when the policyholder is (H)ealthy.

Reserves depends by the attained status (H or I) at the evaluation period.
Empirical data

- We used assumptions for the Italian population taken from (Paolo de Angelis 2016).
- In particular, this exercise is based on the transition probabilities for Italian male population estimated for the 2016 calendar year.
- Possible states are (A)ctive, (I)ll and (D)ead. Modeled transitions are from A -> I, from A -> D and I -> D.
Figure 1: Italian Males Transition Probabilities 2016
The markovchain package

Purpose

- Package created for easily handling Discrete Time Markov Chains (DTMC) in R by S4 classes for homogeneous and not DTMCs.
- Also used to perform structural analysis (e.g. states classification), statistical inference, estimation and simulation.
- The functions written to perform the actuarial analysis on LTC data heavily relies on the markovchain package’s simulation functions.
Package history

- On Cran since mid 2013.
- Core parts written in Rcpp (Eddelbuettel 2013). The simulation function also uses RcppParallel (Allaire et al. 2016).
Application to LTC insurance

- The stochastic process underlying a LTC insurance can be considered a non-homogeneous DTMC, since transition probabilities vary by age.

```r
# defining the transition diagram for age 80
mc80 <- createAgeMarkovChain(80)
mc80

80
A 3-dimensional discrete Markov Chain defined by the following states:
active, ill, dead
The transition matrix (by rows) is defined as follows:

<table>
<thead>
<tr>
<th></th>
<th>active</th>
<th>ill</th>
<th>dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>active</td>
<td>0.9432764</td>
<td>0.02541479</td>
<td>0.03130879</td>
</tr>
<tr>
<td>ill</td>
<td>0.0000000</td>
<td>0.77097391</td>
<td>0.22902609</td>
</tr>
<tr>
<td>dead</td>
<td>0.0000000</td>
<td>0.00000000</td>
<td>1.00000000</td>
</tr>
</tbody>
</table>
```
summarizing the structural proprieties of the transition diagram

summary (mc80)

80 Markov chain that is composed by:
Closed classes:
dead
Recurrent classes:
{dead}
Transient classes:
{active}, {ill}
The Markov chain is not irreducible
The absorbing states are: dead
plot(mc80, main="Transitions for age 80")
The markovchain package allows to draw samples from non-homogenous DTMC.

```r
# simulating life trajectories
table90 <- getTable(age = 90)
simulateLifeTrajectories(transitionTable = table90, numSim = 1)
```

<table>
<thead>
<tr>
<th>Age</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>91</td>
<td>active</td>
</tr>
<tr>
<td>92</td>
<td>active</td>
</tr>
<tr>
<td>93</td>
<td>ill</td>
</tr>
<tr>
<td>94</td>
<td>ill</td>
</tr>
<tr>
<td>95</td>
<td>ill</td>
</tr>
<tr>
<td>96</td>
<td>dead</td>
</tr>
<tr>
<td>97</td>
<td>dead</td>
</tr>
<tr>
<td>98</td>
<td>dead</td>
</tr>
<tr>
<td>99</td>
<td>dead</td>
</tr>
<tr>
<td>100</td>
<td>dead</td>
</tr>
<tr>
<td>101</td>
<td>dead</td>
</tr>
<tr>
<td>102</td>
<td>dead</td>
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<tr>
<td>103</td>
<td>dead</td>
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<tr>
<td>104</td>
<td>dead</td>
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<tr>
<td>105</td>
<td>dead</td>
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<tr>
<td>106</td>
<td>dead</td>
</tr>
<tr>
<td>107</td>
<td>dead</td>
</tr>
<tr>
<td>108</td>
<td>dead</td>
</tr>
<tr>
<td>109</td>
<td>dead</td>
</tr>
<tr>
<td>110</td>
<td>dead</td>
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<tr>
<td>111</td>
<td>dead</td>
</tr>
<tr>
<td>112</td>
<td>dead</td>
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<tr>
<td>113</td>
<td>dead</td>
</tr>
<tr>
<td>114</td>
<td>dead</td>
</tr>
<tr>
<td>115</td>
<td>dead</td>
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<tr>
<td>116</td>
<td>dead</td>
</tr>
<tr>
<td>117</td>
<td>dead</td>
</tr>
<tr>
<td>118</td>
<td>dead</td>
</tr>
<tr>
<td>119</td>
<td>dead</td>
</tr>
<tr>
<td>120</td>
<td>dead</td>
</tr>
<tr>
<td>121</td>
<td>dead</td>
</tr>
<tr>
<td>122</td>
<td>dead</td>
</tr>
</tbody>
</table>
Assumptions

- A simulation approach is used to actuarially evaluate the LTC coverage.
- The (fictionary) policyholder age is 75, the real interest rate is $i = 0.01$.
- We will compute benefit premiums (lump sum and yearly) and reserves.
- A large number of random life trajectories is sampled.
- Cash flows depends by the status at the beginning of the year, 1 when (D)isabled, 0 otherwise.
- The yearly benefit is set to 12K.
Simulating life trajectories

```r
## retrieving transition since age 75
table.75 <- getTable(75)

## simulating life trajectories
lifetrajectories.75 <- simulateLifeTrajectories(transitionTable = table.75,
                                               numSim = 1000,
                                               include_start_age = FALSE,
                                               begin_status = "active")
```
#sampled life trajectories for a policyholder aged 75

lifetrajectories.75[1:5,10:15]

<table>
<thead>
<tr>
<th></th>
<th>85</th>
<th>86</th>
<th>87</th>
<th>88</th>
<th>89</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
</tr>
<tr>
<td>2</td>
<td>&quot;active&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
</tr>
<tr>
<td>3</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
<td>&quot;dead&quot;</td>
</tr>
<tr>
<td>4</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
</tr>
<tr>
<td>5</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
<td>&quot;active&quot;</td>
</tr>
</tbody>
</table>

#computing expected future years disabled

disabled01<-matrix(0,nrow=nrow(lifetrajectories.75),ncol=ncol(lifetrajectories.75))
disabled01[which(lifetrajectories.75=="ill")]=1
mean(rowSums(disabled01))

[1] 1.313
Computing premiums

#PV of a policy paying 12K euro at the beginning of the year
# if the policyholder is disabled
##simulating
pvbenefits.75<-getPVDistribution(lifetrajectoriesMatrix = lifetrajectories.75, target = "ill", CF = 1000*12, real_interest_rate = 0.01, begin = 1)
## computing APV of lump sum benefits

```
U <- mean(pvbenefits.75); U
```

```
[1] 13983.72
```

## yearly premium

```
annuity.75.healthy <- mean(getPVDistribution(lifetrajectoriesMatrix = lifetrajectories.75, target = "active", CF = 1, real_interest_rate = 0.01, begin = 0))
P <- U / annuity.75.healthy; P
```

```
[1] 1383.38
```
It is also possible to perform a stochastic analysis of the future benefits distribution

```r
qplot(pvbenefits.75, main = "LTC benefits PV", xlab = "Euros")
```
Computing reserves

- A prospective approach is used.
- Reserves depend by attained status (Healthy or Ill).
- Calculations performed at example age of 80.
## retrieving transition from age 80
```r
table.80 <- getTable(80)
```

## simulating life trajectories (for healthy insureds)
```r
lifetrajectories.80.healthy <- simulateLifeTrajectories(transitionTable = table.80, 
               numSim = 1000, 
               include_start_age = FALSE, 
               begin_status = "active")
```

## (for ill insureds)
```r
lifetrajectories.80.ill <- simulateLifeTrajectories(transitionTable = table.80, 
               numSim = 1000, 
               include_start_age = FALSE, 
               begin_status = "ill")
```
## benefit reserve distribution, 80 yo healthy

```r
reserve.80.active.distr <-
  getReserveDistribution(lifetrajectoriesMatrix = lifetrajectories.80.healthy,
                         CF_active = -P, CF_ill = +12000, CF_dead = 0, real_interest_rate = 0.01, begin = 0)
mean(reserve.80.active.distr)

[1] 3774.539
```

## benefit reserve distribution, 80 yo ill

```r
reserve.80.ill.distr <-
  getReserveDistribution(lifetrajectoriesMatrix = lifetrajectories.80.ill,
                         CF_active = -P, CF_ill = +12000, CF_dead = 0, real_interest_rate = 0.01, begin = 0)
mean(reserve.80.ill.distr)

[1] 35636.3
```
It is also possible to quantify reserves’ variability
