

# Fairness and Discrimination with Multiple Attributes

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# Motivation

	CA	HI	GA	NC	NY	MA	PA	FL	TX	AL	ON	NB	NL	QC
Gender	×	×	●	×	●	×	×	●	●	●	●	×	×	●
Age	×	×	●	×	●	×	●	●	●	●*	●	×	×	●
Driving experience	●	×	●	●	●	●	●	●	●	●	●	●	●	●
Credit history	×	×	●	●	●	×	●*	●	●	×	×	●*	×	●
Education	×	×	×	×	×	×	●	●	●	●	●	●	●	●
Occupation	×	×	×	●	×	×	●	●	●	●	●	●	●	●
Employment status	×	×	×	●	×	×	●	●	●	●	●	●	●	●
Marital status	●	×	●	●	●	×	●	●	●	●	●	●	●	●
Housing situation	×	×	●	●	●	×	●	●	●	×	×	●	●	●
Address/ZIP code	●	●	●	●	●	●	●	●	●	×	×	●	●	●
Insurance history	●	●	●	●	●	●	●	●	●	●	●	●	●	●

CA: California, HI: Hawaii, GA: Georgia, NC: North Carolina, NY: New York, MA: Massachusetts, PA: Pennsylvania, FL: Florida, TX: Texas, AL: Alberta, ON: Ontario, NB: New-Brunswick, NL: Newfoundland-Labrador, QC: Québec, [The Zebra \(2022\)](#)

```
1 pip install equipy
```

## Notations, risk $\mathcal{R}$ and unfairness $\mathcal{U}$

- ▶  $\mathbf{X} \in \mathcal{X}$ : 'non-sensitive' features,
- ▶  $\mathbf{A} = (A_1, \dots, A_r) \in \mathcal{A}_1 \times \dots \times \mathcal{A}_r$ :  $r$  sensitive features,
- ▶  $\hat{Y}$ : prediction of response variable (continuous or score for a binary classifier)
- ▶  $m$ : predictive model for  $(\mathbf{x}, \mathbf{a})$ , and  $m^*$  the optimal Bayes estimator,
- ▶  $\nu_f$ : distribution of  $m(\mathbf{X}, \mathbf{A})$   
with cumulative distribution function  $F_m$  and quantile function  $Q_m$ ,
- ▶  $\nu_{f|a_i}$ : conditional distribution of  $m(\mathbf{X}, \mathbf{A})|A_i = a_i$  with  $F_{m|a_i}$  and  $Q_{m|a_i}$ ,
- ▶  $\mathcal{R}(m) = \mathbb{E}[(Y - m(\mathbf{X}, \mathbf{A}))^2]$ : (theoretical ) quadratic risk.
- ▶ Given a sample  $\{(y_i, \mathbf{x}_i, \mathbf{a}_i)\}$ , the empirical risk is  $\hat{\mathcal{R}}_n(m) = \sum_{i=1}^n (y_i - m(\mathbf{x}_i, \mathbf{a}_i))^2$
- ▶ Optimal Bayes estimator  $m^* = \operatorname{argmin}\{\mathcal{R}(m)\}$ , is

$$m^*(\mathbf{x}, \mathbf{a}) = \mathbb{E}[Y | (\mathbf{X} = \mathbf{x}, (\mathbf{A} = \mathbf{a}))]$$

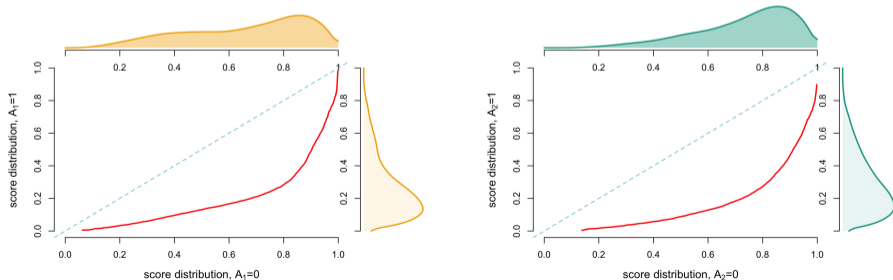
## Notations, risk $\mathcal{R}$ and unfairness $\mathcal{U}$

- ▶ Theoretical quadratic **risk** is  $\mathcal{R}(m) = \mathbb{E}[(Y - m(\mathbf{X}, \mathbf{A}))^2]$
- ▶ Define Wasserstein (2) distance  $\mathcal{W}_2^2(\nu_0, \nu_1) = \min_{Z_0 \sim \nu_0, Z_1 \sim \nu_1} \{\mathbb{E}[(Z_1 - Z_0)^2]\}$

if  $Z_0$  and  $Z_1$  are univariate (+ technical conditions),

$$\mathcal{W}_2^2(\nu_0, \nu_1) = \int_0^1 (Q_1(u) - Q_0(u))^2 du = \mathbb{E}[(Q_1 \circ F_0(Z_0) - Z_0)^2], \quad Z_0 \sim \nu_0$$

$z \mapsto Q_1 \circ F_0(z)$  is the “optimal Monge” transport, [Santambrogio \(2015\)](#)



## Notations, risk $\mathcal{R}$ and unfairness $\mathcal{U}$

- ▶  $m$  is **strongly fair** regarding a **single sensitive attribute** (SSA)  $A_i$ , according to **demographic parity** –  $m(\mathbf{X}, \mathbf{A}) \perp\!\!\!\perp A_i$  – if and only if:

$$\mathcal{U}_i(m) = \max_{a_i \in \mathcal{A}_i} \text{distance}(\nu_m, \nu_{m|a_i}) = 0 \text{ unfairness w.r.t. } A_i$$

- ▶  $m$  is **strongly fair** regarding **multiple sensitive attribute** (MSA), if and only if:

$$\mathcal{U}(m) = \mathcal{U}_1(m) + \dots + \mathcal{U}_r(m) = 0 \text{ unfairness w.r.t. } \mathbf{A}$$

- ▶  $m$  is  **$\epsilon$ -approximately fair** regarding  $A_i$  if and only if  $\mathcal{U}_i(m) \leq \epsilon \cdot \mathcal{U}_i(m^*)$  (where  $m^*$  is the optimal Bayes estimator)

# The Price to Pay for Fairness

- ▶ Let  $\mathcal{M}$  denote a (general) class of models,
- ▶ Let  $\mathcal{M}_{F,i}$  denote the **subset of fair models** with respect to  $A_i$ ,

$$\mathcal{M}_{F,i} = \{m \in \mathcal{M} : \mathcal{U}_i(m) = 0\}$$

- ▶ Fairness is achieved by projection onto the fair subspace

$$\hat{m} \in \operatorname{argmin}_{m \in \mathcal{M}} \{\hat{\mathcal{R}}_n(m)\} \quad \text{and} \quad \hat{m}_{F,i} \in \operatorname{argmin}_{m \in \mathcal{M}_{F,i}} \{\hat{\mathcal{R}}_n(m)\}$$

- ▶ The **price of fairness** of a model class  $\mathcal{M}$

$$\mathcal{E}_{F,i}(\mathcal{M}) = \min_{m \in \mathcal{M}_{F,i}} \{\mathcal{R}(m)\} - \min_{m \in \mathcal{M}} \{\mathcal{R}(m)\} \geq 0.$$

- ▶ **Chzhen et al. (2020)** proved that (+mild technical assumptions)

$$\mathcal{E}_{F,i}(\mathcal{M}) = \min_{\mu \in \mathcal{M}} \left\{ \sum_{a_i \in \mathcal{A}_i} \mathbb{P}(A_i = a_i) \cdot \mathcal{W}_2^2(\nu_{m^*|a_i}, \nu_{\mu|a_i}) \right\}$$

# The Price to Pay for Fairness

- ▶ Given  $K$  distributions  $(\nu_1, \dots, \nu_K)$ , and weights  $(w_1, \dots, w_K) \in \mathbb{R}_+^K$ , the  $\mathcal{W}_2$ -Barycenter is the minimizer:

$$\text{Bar}\{(w_k, \nu_k)_{k=1}^K\} = \underset{\nu}{\operatorname{argmin}} \left\{ \sum_{k=1}^K w_k \cdot \mathcal{W}_2^2(\nu_k, \nu) \right\}.$$

- ▶ SSA ( $r = 1$ ) [Chzhen et al. \(2020\)](#)

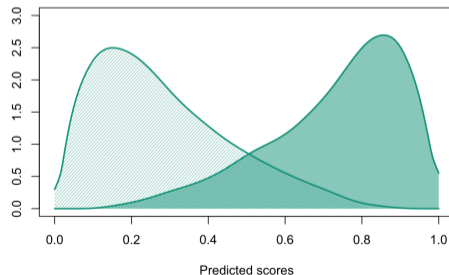
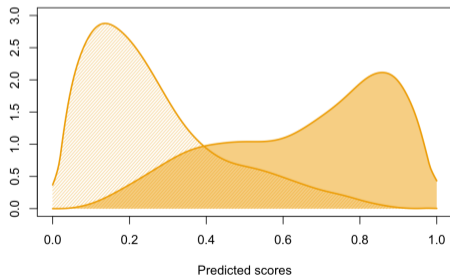
$$\nu_{m_{B_i}} \text{ corresponding to } \min_m \sum_{a_i \in \mathcal{A}_i} \mathbb{P}(A_i = a_i) \cdot \mathcal{W}_2^2(\nu_{m^*|a_i}, \nu_m)$$

$$m_{B_i}(\mathbf{x}, a_i) = \left( \sum_{a' \in \mathcal{A}_i} \mathbb{P}(A_i = a') \cdot Q_{m^*|a'} \right) \circ F_{m^*|a_i}(m^*(\mathbf{x}, a_i)), \quad \forall (\mathbf{x}, a_i) \in \mathcal{X} \times \mathcal{A}_i.$$

→ Equipy: implemented in the function `FairWasserstein` of `fairness` module.

# The Case of Multiple Attributes

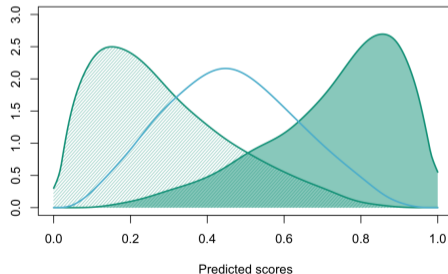
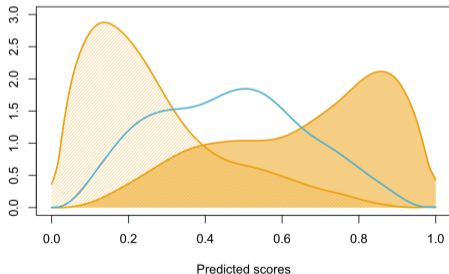
- ▶ Consider a machine Learning model  $m$ , score predictions and two sensitive attributes, ethnic origin  $A_1$  (White/Black) and gender  $A_2$  (Male/Female).
- ▶ Consider densities of  $\nu_{m|A_1=0}$ ,  $\nu_{m|A_1=1}$  (left) and  $\nu_{m|A_2=0}$ ,  $\nu_{m|A_2=1}$  (right)





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- ▶ Plot densities of barycenters,  $\nu_{mB_1}$  and  $\nu_{mB_2}$



# The Case of Multiple Attributes

- ▶ **Intersectional Fairness**, MSA  $\rightarrow$  Single sensitive attribute (SSA), by intersection,

ethnic origin  $A_1$   
gender  $A_2$

$$\mathbf{a} \in \mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 = \{\text{white, black}\} \times \{\text{male, female}\}$$

Here  $\mathcal{A}$  corresponds to  $4 = 2 \times 2$  states,

$$\mathcal{A} = \{(\text{white, male}), (\text{white, female}), (\text{black, male}), (\text{black, female})\}$$

- ▶ **Sequential Fairness**, MSA, in [Hu et al. \(2024\)](#)

# The Case of Multiple Attributes

- ▶ MSA ( $r \geq 1$ ) [Hu et al. \(2024\)](#)

$$m_B(\mathbf{x}, \mathbf{a}) := m_{B_1} \circ m_{B_2} \circ \cdots \circ m_{B_r}(\mathbf{x}, \mathbf{a})$$

where

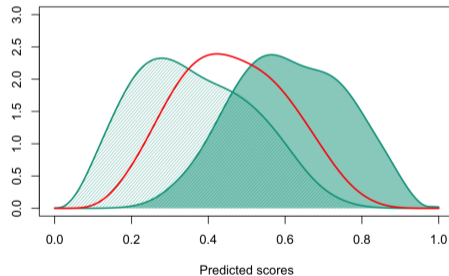
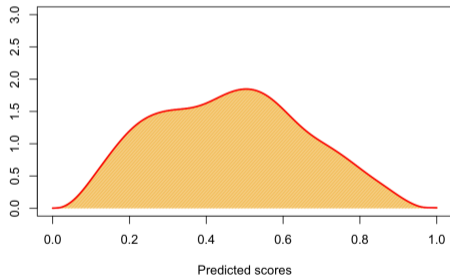
$$m_{B_i} \circ m_{B_j}(\mathbf{x}, \mathbf{a}) = \left( \sum_{a' \in \mathcal{A}_i} \mathbb{P}(A_i = a') Q_{m_{B_j}|a'} \right) \circ F_{m_{B_j}|a_i}(m_{B_j}(\mathbf{x}, \mathbf{a}))$$

$\forall (\mathbf{x}, \mathbf{a}) \in \mathcal{X} \times \mathcal{A}_{1:r}$ , with the  $i$ -th component of  $\mathbf{a}$  denoted  $a_i$ .

- ▶ [Hu et al. \(2024\)](#) proved the **associativity of Wasserstein barycenters** (fairness mitigation remains unaffected by the order of  $A_{1:r}$ ).
- EquiPy: implemented in the function `MultiWasserstein` of `fairness` module.

# The Case of Multiple Attributes

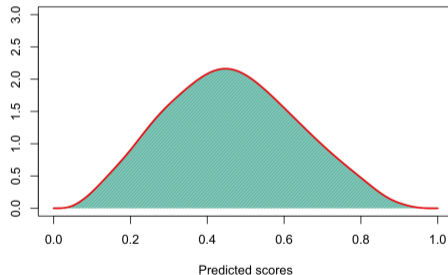
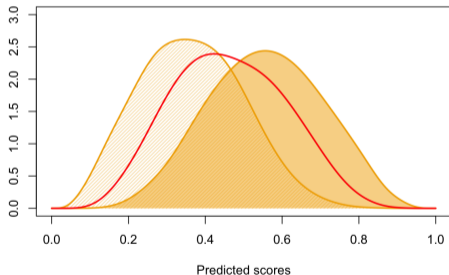
- ▶ Given  $\nu_{m_{B_1}}$ , consider
  - ▶ the barycenter  $\nu_{m_{B_1}}$  conditional on  $A_1$  (no impact, already fair)
  - ▶ the barycenter  $\nu_{m_{B_2}}$  conditional on  $A_2$



- ▶ On the right, distribution of  $\nu_{m_{B_2} \circ m_{B_1}}$

# The Case of Multiple Attributes

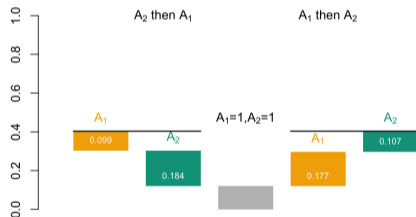
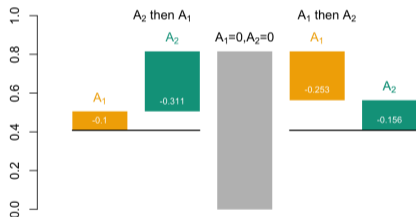
- ▶ Given  $\nu_{m_{B_2}}$ , consider
  - ▶ the barycenter  $\nu_{m_{B_1}}$  conditional on  $A_1$
  - ▶ the barycenter  $\nu_{m_{B_2}}$  conditional on  $A_2$  (no impact, already fair)



- ▶ On the left, distribution of  $\nu_{m_{B_1} \circ m_{B_2}}$

# The Case of Multiple Attributes

- ▶ The order of this **sequential approach** leads different interpretations,
  - ▶ left hand part,  $A_2$  then  $A_1$
  - ▶ right hand part,  $A_1$  then  $A_2$



# Life insurance dataset

- ▶ Public SEER database: <https://seer.cancer.gov>,
  - ▶ Prediction of **one-year mortality** of US individuals with melanoma skin cancer,  
→ Use the methodology presented in [Sauce et al. \(2023\)](#), we convert the dataset into survival data, by accounting for exposure over a given time interval.
  - ▶ Sample size  $n = 547,878$  from 2004 to 2018,
  - ▶ Explanatory variables: 16 features describing patient characteristics (age, **gender** male/female, **ethnic origin**) and cancer attributes (tumor size, extent).
- MSA framework: use of the function `MultiWasserstein`.

# Model fitting

1. Split the data into train and test sets,
2. Fit Logistic Regression  $m$  ( $m$  can be any ML model, model-agnosticity)
3. Apply  $m$  on the test set to obtain  $\hat{y}_{\text{test}}$ .

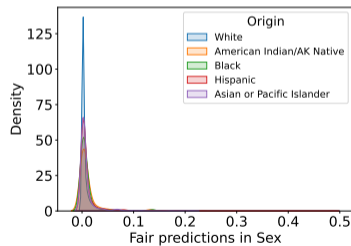
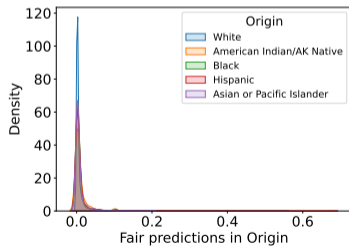
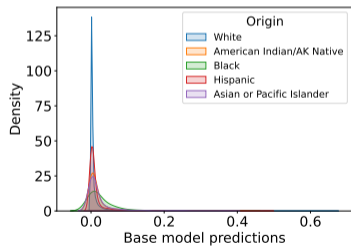
We consider different model fitting scenarios, in which we include or exclude sensitive attributes as explanatory variables:

Ethnic origin	Gender	AUC	Unfairness
No	No	0.8652	0.2179
<b>Yes</b>	<b>Yes</b>	0.8672	0.2668



# Transforming predictions

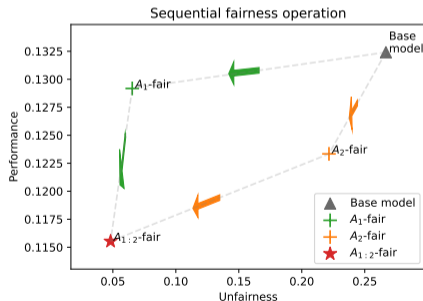
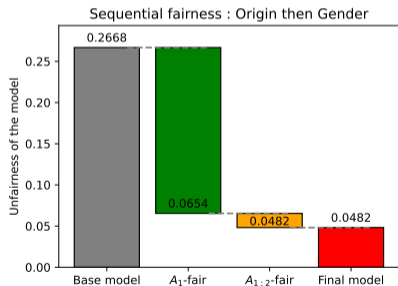
1. Split the test data into calibration and test sets,
2. Specify an order to sequentially correct:  $A_1$  corresponds to ethnic origin and  $A_2$  corresponds to gender,
3. Fit and transform your test predictions using `MultiWasserstein` from `fairness` module.



# Visualizations

Unfairness and metric calculations with `graphs` module:

- ▶ `fair_waterfall_plot`: sequential gain in fairness for the specified order  $A_1$  then  $A_2$ ,
- ▶ `fair_multiple_arrow_plot`: fairness-performance relationship for all potential pathways.



## Additional results: Approximate fairness

When correcting biases related to **gender**, we reduce fairness regarding **origin**:

Fairness step	Unfairness in <b>origin</b>	Unfairness in <b>gender</b>
Base model	<b>0.2371</b>	0.0297
<b>Origin</b>	<b>0.0345</b>	0.0309
<b>Origin &amp; Gender</b>	<b>0.0469</b>	0.0013

We can prioritize fairness across attributes by specifying  $\epsilon = [0, 0.5]$  corresponding to exact fairness in  $A_1$  and 0.5-approximate fairness in  $A_2$ .

$$m_B = 0.5 \cdot (m_{B_2} \circ m_{B_1}) + 0.5 \cdot m_{B_1}$$

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