# A multi-class problem perspective

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**Summary** Algorithmic Fairness with multi-class problem perspective



## 2) Approximate fairness

3) Some results in Insurance



### **Problem formulation**

Fairness in multi-class problem with demographic parity



Increasingly prevalent in actuarial studies

Illustrative example



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#### Increasingly prevalent in actuarial studies

Illustrative example



**FREQUENCIES** 



#### Increasingly prevalent in actuarial studies





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#### **Fairness in multi-class classification problem**

Demographic Parity measure of fairness

Observations:
$$(\underbrace{\text{features}}_X, \underbrace{\text{sensitive attribute}}_S, \underbrace{\text{label}(s)}_Y)$$
(Misclassification) Risk: $\mathcal{R}(g) = \mathbb{P}(g(X, S) \neq Y)$ Scores: $p_k(X, S) = \mathbb{P}(Y = k \mid X, S)$ Bayes classifier: $g^* \in \arg \min_g \left\{ \mathcal{R}(g) \right\} \triangleright g^*(x, s) \in \arg \max_k p_k(x, s)$ 



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### **Exact and Approximate Fairness**

Optimal fair predictor and statistical guarantees



#### **Exact and approximate fairness**

Risk: $\mathcal{R}(g) = \mathbb{P}(g(X, S) \neq Y)$ Unfairness measure: $\mathcal{U}(g) = \max_k | \mathbb{P}(g(X, S) = k | S = 1) - \mathbb{P}(g(X, S) = k | S = -1) |$ Exact fairnessApproximate (or  $\varepsilon$ ) fairness $\mathcal{U}(g) = 0$  $\mathcal{U}(g) \leq \varepsilon$ 



#### **Method of Langrange multipliers**

 $\begin{array}{lll} \mbox{Risk:} & \mathcal{R}(g) = \mathbb{P}(g(X,\,S) \neq Y) \\ & \mbox{Unfairness measure:} & \mathcal{U}(g) = \max_k | \mathbb{P}(g(X,S) = k \mid S = 1) - \mathbb{P}(g(X,S) = k \mid S = -1) | \\ & \swarrow & & & \\ \hline & \mbox{Exact fairness} & \mbox{Approximate (or } \mathcal{E} \mbox{) fairness} \\ & \mathcal{U}(g) = 0 & & \mathcal{U}(g) \leq \varepsilon \end{array}$ 

#### Fair-riskLagrangian of the problem

$$\begin{aligned} \mathcal{R}_{\lambda^{(1)},\lambda^{(2)}}(g) &:= \mathcal{R}(g) + \sum_{k=1}^{K} \lambda_k^{(1)} [\mathbb{P}\left(g(X,S) = k | S = 1\right) - \mathbb{P}\left(g(X,S) = k | S = -1\right) - \varepsilon \right] \\ &+ \sum_{k=1}^{K} \lambda_k^{(2)} [\mathbb{P}\left(g(X,S) = k | S = -1\right) - \mathbb{P}\left(g(X,S) = k | S = 1\right) - \varepsilon ] \end{aligned}$$

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#### **Optimal fair prediction and statistical guarantees (\*)**

#### **Theorem (informal)**

$$\begin{aligned} & \text{If} \qquad (\lambda^{*(1)}, \lambda^{*(2)}) \in \arg\min_{(\lambda^{(1)}, \lambda^{(2)}) \in \mathbb{R}^{2K}_+} \sum_{s \in \mathcal{S}} \mathbb{E}_{X|S=s} \left[ \max_k \left( \pi_s p_k(X, s) - s(\lambda_k^{(1)} - \lambda_k^{(2)}) \right) \right] + \varepsilon \sum_{k=1}^{\kappa} (\lambda_k^{(1)} + \lambda_k^{(2)}) \\ & \text{then} \quad g^*_{\varepsilon-\text{fair}}(x, s) = \arg\max_{k \in [K]} \left( \pi_s p_k(x, s) - s(\lambda_k^{*(1)} - \lambda_k^{*(2)}) \right) \end{aligned}$$



#### **Closed-form** solution



#### Post-processing and model agnostic

**Theorem (informal)**: a plug-in estimator makes the model asymptotically as performant as  $g_{\varepsilon-\text{fair}}^*$  in terms of **fairness** and **predictive performance** 

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### Numerical evaluation (\*)

Dataset: DRUG, CRIME

Machine Learning Models:
Logistic regression (GLM)
Random Forest (RF)
LightGBM (GBM)

Benchmark: Fair-projection (1)



(\*) Our paper: fairness guarantees in multi-class classification with demographic parity (JMLR 2024)

(1) Baseline: post-processing approach for multi-classes (NeurIPS 2022)

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### **Insurance dataset**

Car insurance portfolio



#### **Description of the dataset**

Toy example: automobile insurance portfolio

#### Real-world example

The **freMPL** dataset (CASdataset) is a database used in the automobile insurance industry. It contains information on driver characteristics, insured vehicles and associated claims (+10k observations).





#### Numerical evaluation (1/2)

Toy example: automobile insurance portfolio



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#### **Numerical evaluation (2/2)**

Toy example: automobile insurance portfolio





### In summary

- Multi-class classification paradigm enables precise risk categorization, enhancing accuracy in insurance pricing.
- Post-processing approach applicable to any off-the-shelf ML model
- Compared to regression tasks, multi-class framework can better achieve other fairness metrics like separation and sufficiency.





### Thank you for your attention !

